Related-Key Attack on Full-Round PICARO

SAC 2015

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Inria, France

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Outline

1. PICARO
2. Keys Leading to Colliding Ciphertexts
3. Related-Key Attack
4. Conclusion

$$\begin{tikzcd} P \arrow[r, "E_K"] \arrow[r, bend right, "E_{K\oplus\Delta}"'] & C \end{tikzcd}$$

PICARO

Gilles Piret, Thomas Roche and Claude Carlet
PICARO - A Block Cipher Allowing Efficient Higher-Order Side-Channel Resistance,
ACNS 2012.

Background

Objective

Build a cipher that would be easy to protect against side-channel attacks

All countermeasures have a high performance overhead

$\rightarrow$ Start from the masking scheme, determine the parts that are hard to mask and then design the cipher accordingly

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PICARO is more efficient than AES when masked using Rivain-Prouff's scheme

PICARO

Focus on the Sbox, with special care to:

- non-linearity
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- algebraic degree
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$$\begin{aligned} S : GF(2^4) \times GF(2^4) &\rightarrow GF(2^4) \times GF(2^4) \\ (x, y) &\mapsto (xy, (x^3 + \mathtt{0x02})(y^3 + \mathtt{0x04})) \end{aligned}$$

Claude Carlet
Relating Three Nonlinearity Parameters of Vectorial Functions and Building APN functions from Bent Functions,
Designs, Codes and Cryptography 2011.

PICARO

Focus on the Sbox, with special care to:

- non-linearity $nl = 94$
- maximal differential probability $\delta = 4/2^8$
- algebraic degree $d = 4$
- ease to mask 4 non-linear operations in $GF(2^4)$

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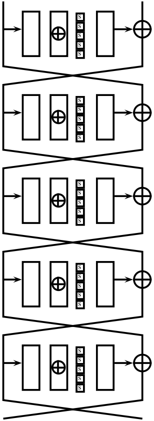
Non-Bijective

Round Function: Possible Threat



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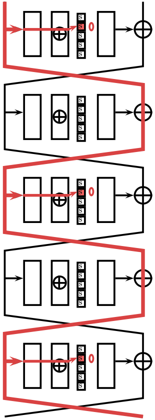
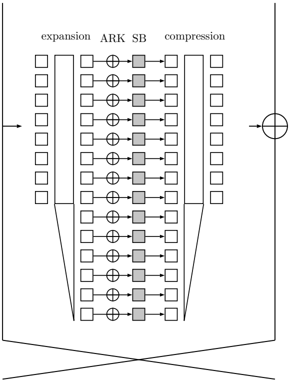


- Possible to have only 1 round active out of 2 with only 1 active Sbox
- Need to ensure a minimum number of active Sboxes per round

Round Function

Solution Proposed: Expansion and Compression layers



12 rounds

- expansion from 8 bytes to 8+6 bytes
- key addition
- Sbox layer
- compression from 8+6 bytes back to 8 bytes

MDS code [8+6, 8, 7] of generator matrix:

$$G = \begin{pmatrix} Id_8 & \mathscr{G} \end{pmatrix}$$

with $\mathscr{G} = \begin{pmatrix} \texttt{01} & \texttt{01} & \texttt{0A} & \texttt{01} & \texttt{09} & \texttt{0C} \\ \texttt{05} & \texttt{01} & \texttt{01} & \texttt{0A} & \texttt{01} & \texttt{09} \\ \texttt{06} & \texttt{05} & \texttt{01} & \texttt{01} & \texttt{0A} & \texttt{01} \\ \texttt{0C} & \texttt{06} & \texttt{05} & \texttt{01} & \texttt{01} & \texttt{0A} \\ \texttt{09} & \texttt{0C} & \texttt{06} & \texttt{05} & \texttt{01} & \texttt{01} \\ \texttt{01} & \texttt{09} & \texttt{0C} & \texttt{06} & \texttt{05} & \texttt{01} \\ \texttt{0A} & \texttt{01} & \texttt{09} & \texttt{0C} & \texttt{06} & \texttt{05} \\ \texttt{01} & \texttt{0A} & \texttt{01} & \texttt{09} & \texttt{0C} & \texttt{06} \end{pmatrix}$

Keys Leading to Colliding Ciphertexts

Preliminary Remarks

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Any entering difference has a probability of 2^{-7} of being cancelled

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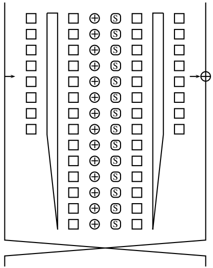

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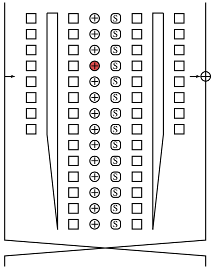

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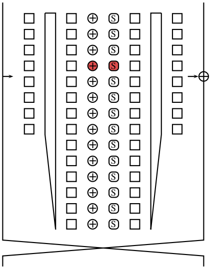

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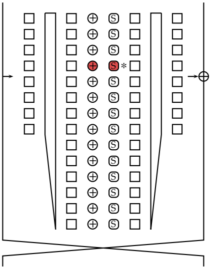

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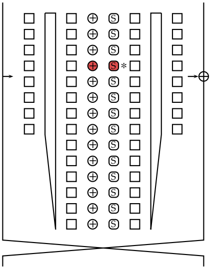

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How far can we go?

Keys Leading to Colliding Ciphertexts

Question:
Can we find a master key difference Δ such that for random (P, K) we have with high probability $E_K(P) = E_{K \oplus \Delta}(P)$?

To cancel all the key differences, we can afford a maximum of s Sbox cancellations, with s satisfying:

$$2^{-7s} > 2^{-128}$$

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$\rightarrow$ Find a Master Key difference that activates less than 18 bytes in the subkeys

Key Schedule

- Master key K of 128 bits
- 12 round-keys k^i of 112 bits

$$\begin{cases} \kappa^1 = K \\ \kappa^i = T(\kappa^{i-1}) \ggg \theta(i) & \text{for } i = 2, \cdots, 12 \end{cases}$$

$$\begin{pmatrix} T(K)^{(1)} \\ T(K)^{(2)} \\ T(K)^{(3)} \\ T(K)^{(4)} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \times \begin{pmatrix} K^{(1)} \\ K^{(2)} \\ K^{(3)} \\ K^{(4)} \end{pmatrix}$$

where θ is defined by:

i	2	3	4	5	6	7	8	9	10	11	12
$\theta(i)$	1	15	1	15	1	52	1	15	1	15	1

k^i = first 112 bits of κ^i

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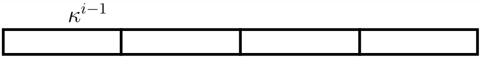

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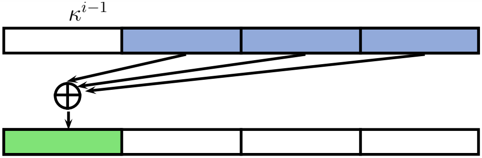

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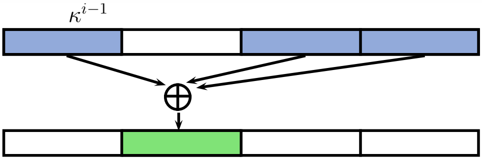

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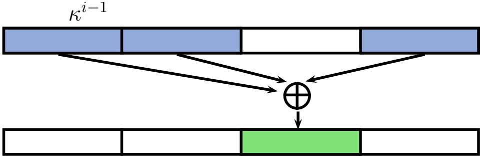

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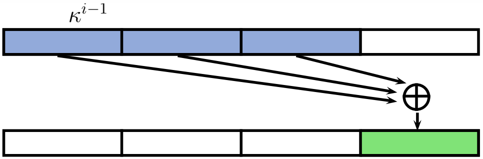

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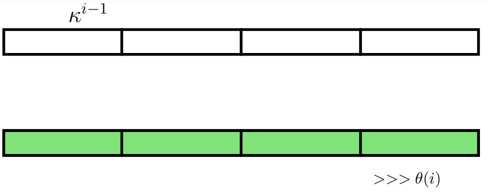

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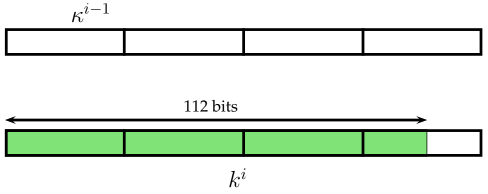
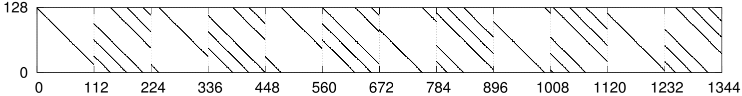


Keys Leading to Colliding Ciphertexts

Key Schedule totally linear over $GF(2)$ $\Leftrightarrow$ linear code



First approximation: look for low-weight codewords (in bits)

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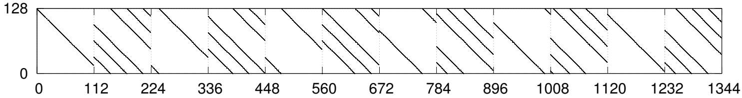

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Remark: Each master key bit flipped results in a minimum of 4 bits flipped in the odd round subkeys (k_1, k_3, k_5, k_7, k_9, k_{11})

$\rightarrow$**Codewords of weight** ≤ 18 **obtained by exhausting all master keys of weight** ≤ 4

Keys Leading to Colliding Ciphertexts

Minimum distance 18

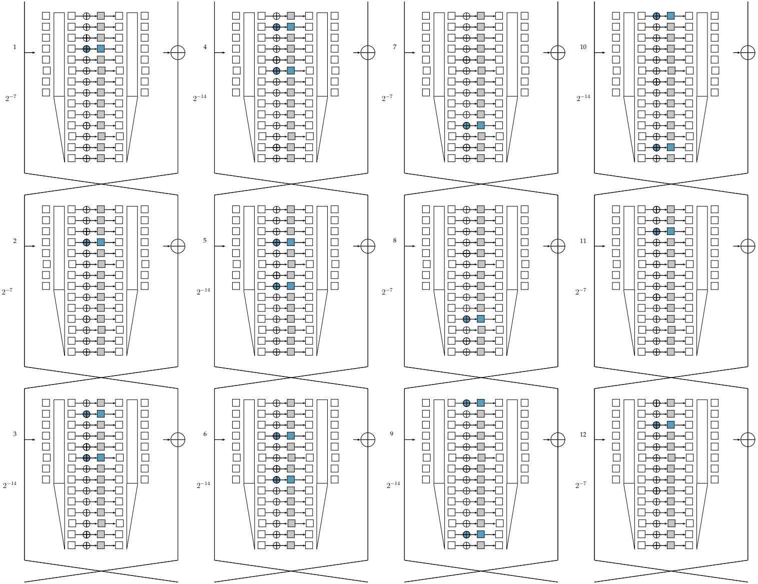
8 words/master key differences reaching that minimum:

config.	K	k^1	k^2	k^3	k^4	k^5	k^6	k^7	k^8	k^9	k^{10}	k^{11}	k^{12}
1	27, 123	27	28	11, 43	12, 44	27, 59	28, 60	80	81	0, 96	1, 97	16	17
2	28, 124	28	29	12, 44	13, 45	28, 60	29, 61	81	82	1, 97	2, 98	17	18
3	29, 125	29	30	13, 45	14, 46	29, 61	30, 62	82	83	2, 98	3, 99	18	19
4	30, 126	30	31	14, 46	15, 47	30, 62	31, 63	83	84	3, 99	4, 100	19	20
5	91, 123	91	92	11, 107	12, 108	27	28	48, 80	49, 81	64, 96	65, 97	80	81
6	92, 124	92	93	12, 108	13, 109	28	29	49, 81	50, 82	65, 97	66, 98	81	82
7	93, 125	93	94	13, 109	14, 110	29	30	50, 82	51, 83	66, 98	67, 99	82	83
8	94, 126	94	95	14, 110	15, 111	30	31	51, 83	52, 84	67, 99	68, 100	83	84

Byte distance: minimum of 18 active bytes
30 words/master key differences reaching that minimum

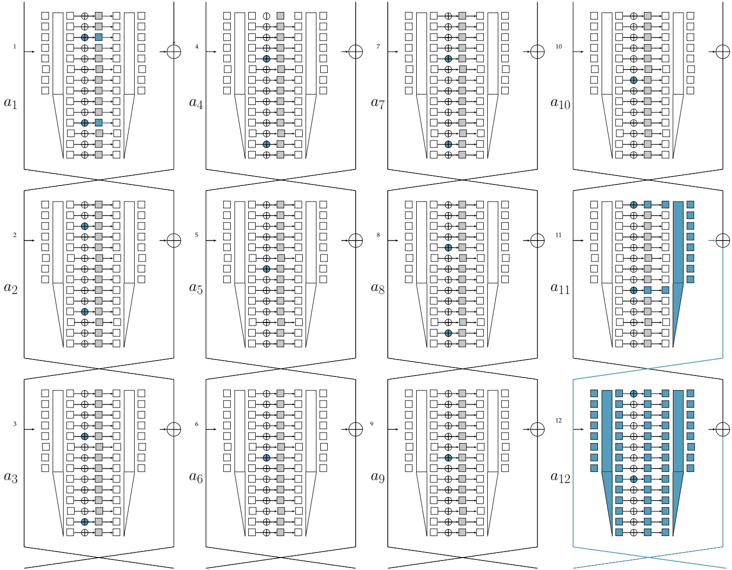
$\rightarrow$ Ciphertexts collide with probability $2^{-18\times 7} = 2^{-126}$

Distinguisher



Related-Key Attack

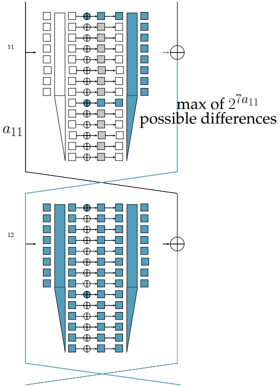
Idea: Mounting a 2R-attack



Properties

Ciphertext Filter

For a plaintext and a pair of keys following the characteristic, only $2^{7\times a_{11}}$ differences are possible out of 2^{64} for the right half of the ciphertext



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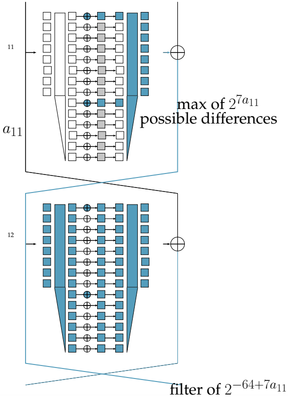

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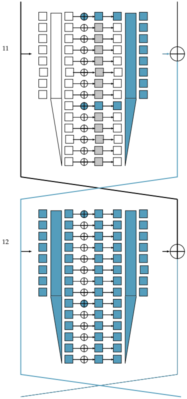
Compression Function Property (for values and differences)

The knowledge of the output of the compression function and of any 6 bytes of the input is sufficient to uniquely determine all input bits



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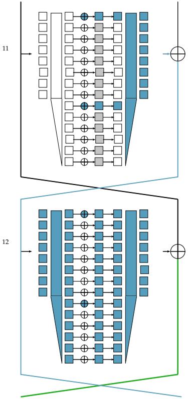

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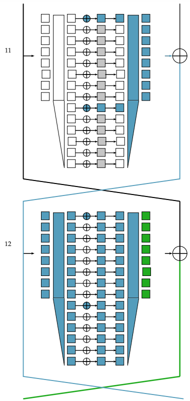

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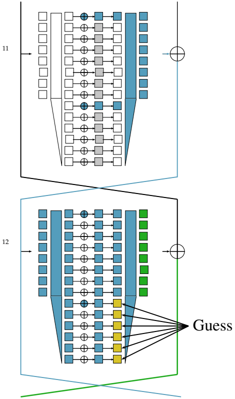

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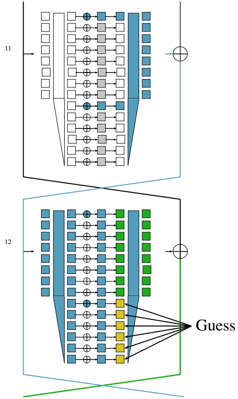
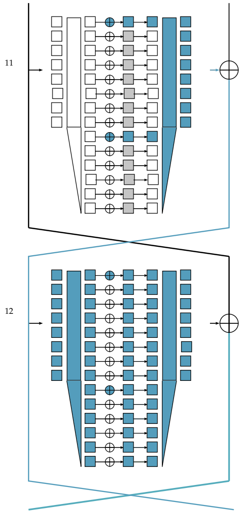


Basic Attack



1. Ask for $2^{7a_1 \to a_{10}}$ messages encrypted with both K and $K \oplus \Delta$

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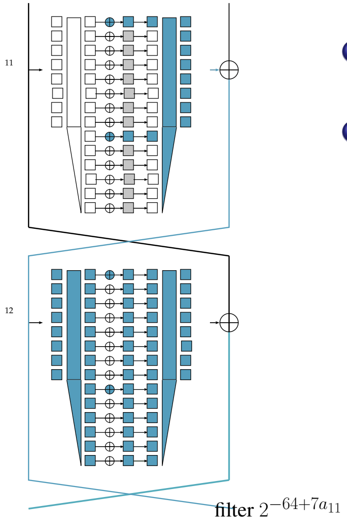

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2. Filter out the pairs without the correct ciphertext difference

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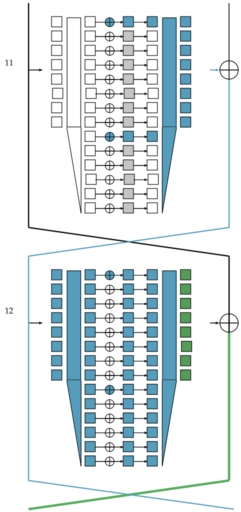

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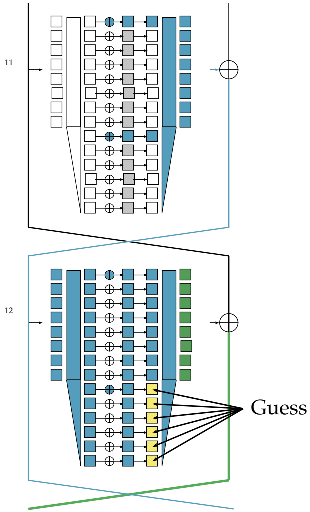

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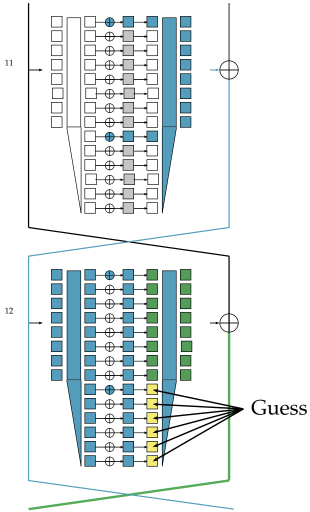

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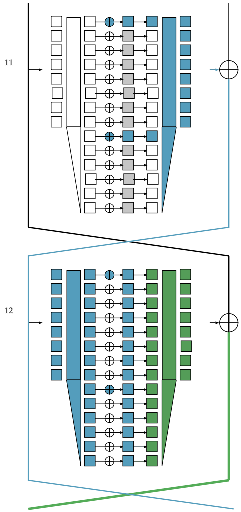

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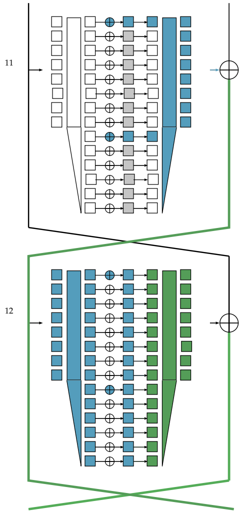

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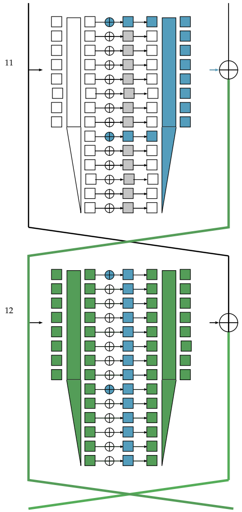

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4. Compute expansion function in differences and add Δ

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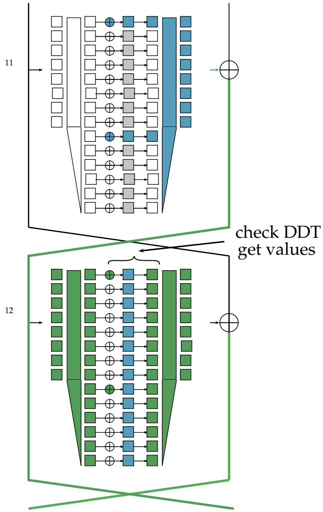

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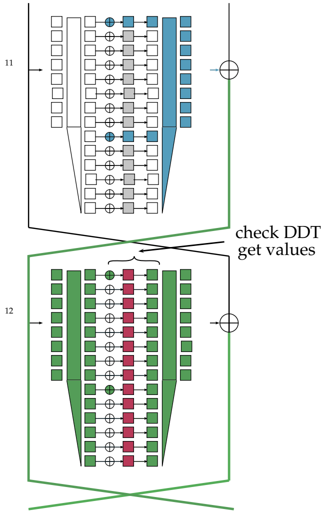

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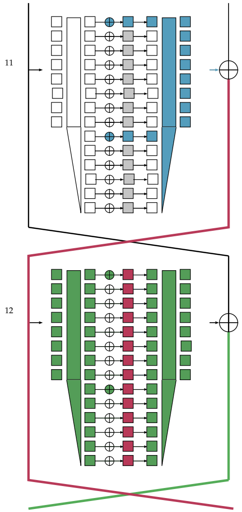

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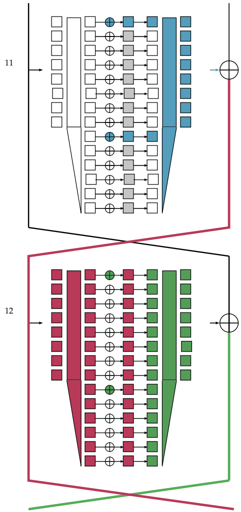
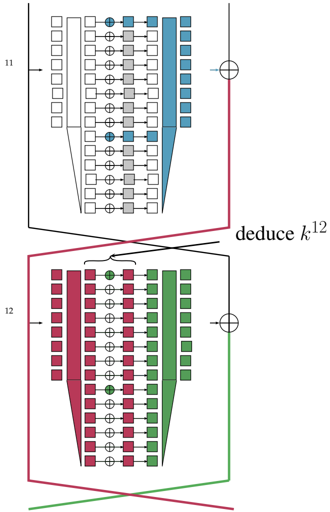
5. Check the DDT and deduce values
6. With ciphertext value, deduce k^{12}

Basic Attack



1. Ask for $2^{7a_1 \to a_{10}}$ messages encrypted with both K and $K \oplus \Delta$
2. Filter out the pairs without the correct ciphertext difference
3. Guess 6-byte differences and deduce whole *Comp* input difference
4. Compute expansion function in differences and add Δ
5. Check the DDT and deduce values
6. With ciphertext value, deduce k^{12}
7. Use previous rounds to filter out keys

Basic Attack



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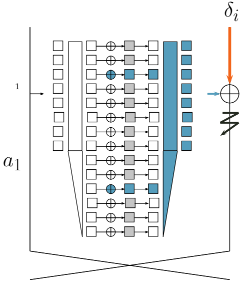
Improvement: Structure-like Technique

Let the first round-key difference spreads freely and cancel it with a plaintext difference introduced in right hand plaintext half



Encrypt the 2^{8a_1} messages $P \oplus \delta$ under the keys K and under $K \oplus \Delta$ where the δ covers all the possible differences at the output of the compression function

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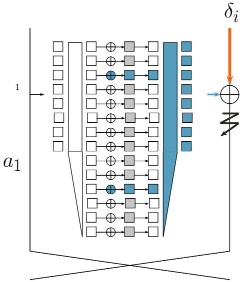

Encrypt the 2^{8a_1} messages $P \oplus \delta$ under the keys K and under $K \oplus \Delta$ where the δ covers all the possible differences at the output of the compression function

$2^{8a_1} + 2^{8a_1} = 2^{8a_1+1}$ encryptions
give $2^{8a_1} \times 2^{8a_1} \times 2^{-8a_1} = 2^{8a_1}$ pairs that pass the first round conditions

$$\rightarrow 2^{7 \times a_{2\rightarrow 10}+1} \text{ encryptions in total (vs } 2^{7 \times a_{1\rightarrow 10}+1} \text{)}$$

Choosing Parameters

Memory:

$$2^{8\times a_1+1}$$

Data:

$$2^{7\times a_{2\to 10}+1}$$

Time:

$$2^{7\times a_{2\to 10}+1}+2^{7\times a_{1\to 11}-18.58}$$

$a_{2\to 10}$	$a_{1\to 11}$	Memory	Data	Time
14	18	2^{17}	2^{99}	$2^{107.4}$
15	17	2^{9}	2^{106}	2^{106}

Conclusion

Conclusion

- While the designers targeted resistance against related-key attacks, we have shown a full-round cryptanalysis of PICARO under this model
- The main weakness exploited here (and one that should be fixed) is the small diffusion of its key schedule, which turns out to be devastating when combined with the non-bijective Sboxes

Thank you for your attention