Dry friction models for automatic control

P.-A. Bliman, M. Sorine
INRIA, Rocquencourt BP 105, 78153 Le Chesnay cedex, France

Introduction:

Dry friction modeling for simulation and control of mechanical systems is the object of an intense activity in Control and Robotics [6].

Usually, the friction limits the performances (precision, stability). Models are then needed to estimate this limitation by simulations, and to compensate in real-time for the friction. This is the case e.g. for path following for robots, line of sight stabilization for pointing devices, ... A high friction coefficient may also be needed, e.g. at the contact between the tyres of a vehicle and the road. Models are then useful to simulate the behavior of the vehicle, and to control this behavior (Anti lock Braking Systems (ABS), steering control systems, estimation of tyre/ground friction, ...) [1, 2].

We propose a class of dry friction models, having the following properties:

• **Agreement with experiments:** The models have the so-called *hysteresis property*[1]: in the case of a periodic motion, friction-displacement cycles appear, that do not depend upon the velocity. These properties, namely *rate-independence* and *memory effect*, are important to insure model validity in a large range of velocities, and especially at low speed. The analogy between friction-displacement and stress-strain relations leads to interpret these models as *elastoplastic* ones [3].

The models are *dissipative*. Though, they may present local maxima of the friction (the *static friction*) for some breakaway distance from rest, before an asymptotic value (the *kinetic friction*) is attained if the velocity is not reversed for a long enough distance. This rate-independent static to kinetic transition, when it occurs, gives rise to locally negative spring stiffness, which may lead to autonomous oscillations, the so called *stick-slip limit cycles* observed in some controled devices.

• **Well-posedness:** When coupled with the equations of motion, the models lead to well-posed problems, a property necessary to derive sound numerical schemes.

• **Simplicity:** The models are simple enough to permit real-time identification and use in on-board systems. They are defined by Ordinary Differential Equations, easy to use compared to usual models for hysteresis.

The proposed models:

The models we propose [1, 2] appear as the filtering of the multivalued Coulomb friction model $\operatorname{sgn} \dot{u}$ (u being the displacement) by linear differential operators based upon an "*intrinsic time*", as in the endochronic theory of material behaviors [4]. In order to satisfy the rate-independence property, this "time" is the curvilinear abscissa s of the trajectory: the friction f appears as a solution of $T(\frac{d}{ds})f = \frac{du}{ds}$ for a rational proper and stable transfer function $T(p) = C(pI - A)^{-1}B + D$ (p is the Laplace variable associated to s):

$$\dot{x} = |\dot{u}| \cdot Ax + B\dot{u}, \quad f(u)(t) = Cx(t) + D \operatorname{sgn} \dot{u}, \quad x(0) = 0, \quad x \in \mathbb{R}^n \tag{1}$$

[1] Following [5], we define an hysteresis operator $f(u)$ as a rate-independent causal operator with memory.

Coulomb model corresponds to $f(u) = D\frac{du}{ds}$ (zero-order model). In the sequel, we suppose $D = 0$, corresponding to smoothed friction model. A 1st order model permits to obtain the elastic and plastic behaviors, a 2nd order model permits to describe static friction too; both are dissipative [1, 2]. The well-posedness of the equations of motion is derived from a local Lipschitz property of the hysteresis operator $u \mapsto f(u)$ in adequate space. The linear representation wrt the intrinsic time ensures the required simplicity.

Thermomechanical interpretation:

The dissipativity property of the models relies on the *positivity* of the transfer function T. Simple matrix criteria permit to ensure this property[2]. As an example, such a criterion is: $\exists P = P^T > 0, Q = Q^T > 0, A^TP + PA = -Q, C^T = PB$, and the power $f\dot{u}$ then writes: $f\dot{u} = \frac{1}{2}(Qx,x)|\dot{u}| + \frac{1}{2}\frac{d}{dt}(Px,x)$. This leads to the thermomechanical interpretation: $\mathcal{D} \triangleq \frac{1}{2}(Qx,x)|\dot{u}|$ is a *pseudo-potential of dissipation* and $\mathcal{F} \triangleq \frac{1}{2}(Px,x)$ a *free energy.*

The classical above-mentioned analogy between friction and plasticity leads to some kind of decomposition of the "deformation" u into an *elastic* part u_e and a *plastic* part u_p. Making the change of variable $u_P \triangleq Bu - x$ (and $u_e = x$), (1) writes

$$\dot{u}_p = |\dot{u}|A(u_P - Bu), \quad f(u)(t) = B^TP(Bu - u_P) \quad \text{with } PA^{-1} + A^{-T}P = -I$$

(taking $Q = A^TA$) and the power $f\dot{u}$ then writes:

$$f\dot{u} = \frac{1}{2}\left\|\frac{\dot{u}_p}{|\dot{u}|}\right\|^2 |\dot{u}| + \frac{1}{2}\frac{d}{dt}(P(Bu - u_p), (Bu - u_p))$$

Wrt the intrinsic time s, $\mathcal{D}$ becomes $\mathcal{D}_s = \frac{1}{2}\left\|\frac{du_p}{ds}\right\|^2$ (and $\mathcal{D}\cdot dt = \mathcal{D}_s \cdot ds$), where u_p is obtained by filtering of u: $\frac{du_p}{ds} = Au_p - Bu$. One has $f(u) = \frac{\partial \mathcal{F}_s}{\partial u}$.

[1] Bliman P.-A., Sorine M., Easy-to-use realistic dry friction models for automatic control, *Proc. of the 3rd Eur. Cont. Conf.*, Roma, Italy, 3788-3794, 1995

[2] Bliman P.-A., Bonald T., Sorine M., Hysteresis operators and tyre friction models. Application to vehicle dynamic simuilation, to appear in the Proc. of ICIAM 1995, help in Hamburg, 3-7 July

[3] Haupt P., On the mathematical modelling of material behavior in continuum mechanics, *Acta Mechanica*, 100, 129-154, 1993

[4] Valanis K.C., A theory of viscoplasticity without a yield surface, *Archives of Mechanics*, Warszawa, **23**, 4, 1971

[5] Visintin A., *Mathematical models of hysteresis*, Topics in nonsmooth analysis, Birkhaüser Verlag, Basel Boston Berlin, 295-326, 1988

[6] Friction-induced vibration, R.A. Ibrahim, E. Rivin eds, *Applied Mech. Reviews*, **47**, 7, 208-305, 1994

[2] They may be expressed wrt the internal or external representation of $f(u)$, leading to a spectral interpretation of dissipativity: $\forall \omega \in \mathbb{R}$, Re $T(j\omega) \geq 0$.