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Hysteresis Operators and Tyre Friction Models. Application to Vehicle Dynamic Simulation

We propose a class of dry friction models for simulation and control of mechanical systems. We apply it to tyre/road contact modeling and show how to use it for vehicle dynamic simulation and control.

1. Introduction

Dry friction modeling for simulation and control of mechanical systems is the subject of an intense activity in Control science and Robotics ([2] for a review). Models of the tyre/road contact are useful to simulate vehicle behavior [10,9,1] or to elaborate e.g. Anti-lock Braking Systems or steering control systems. To make up such models, we use the mathematical theory of the systems with hysteresis, widely spread in the last decade [12,19,4]. We propose a class of hysteresis operators defined by ODEs [4,5]. We require that **1.** They are hysteretic: in the case of periodic motions, one observes experimentally that the hysteresis cycles, friction vs. relative displacement, do not depend upon the velocity, but upon the covered distance. This property is especially important to obtain good models at low velocities. **2.** They are *dissipative*, even though there may exist local maxima of the friction or lag in the friction/position relation. **3.** They lead to well-posed problems when coupled to equations of motions. This is essential for numerical simulations. **4.** They are simple enough to be used in real-time in on-board systems.

2. Classical dry friction models

We consider only systems with single DOF, a generalized displacement u. The simpler model is Coulomb model:

$$F(u) = \begin{cases} f_k & \text{pour} \quad \dot{u} > 0 \\ [-f_s, f_s] & \text{pour} \quad \dot{u} = 0 \\ -f_k & \text{pour} \quad \dot{u} < 0 \end{cases} \tag{1}$$

$F(u)$ may take any value inside $[-f_s, f_s]$ when $\dot{u} = 0$ to balance the external forces: F is multivalued. f_s (resp. f_k) is the *static* (resp. *kinetic*) friction coefficient. For the usual Coulomb model, $f_s = f_k$. Model (1) has major drawbacks. It provides very few informations on $F(u)$ when $\dot{u}$ crosses 0: only bounds. When one regulates u, $\dot{u}$ oscillates around 0, so the behavior of $F(u)$ around $\dot{u} = 0$ is fundamental. Moreover, (1) leads to consider the equation of motion as a differential inclusion [8,13], ill-posed when $f_s > f_k$ (no uniqueness) and requiring regularization in $\dot{u} = 0$ to be solved numerically. The simulated transients will depend upon the choice of the regularization.

A precise analysis of the solid/solid dry friction with stiction [2,7,16] shows an *elastic behavior* (Dahl effect) after departure from rest and until distance s_e is covered, for which the friction is maximal and equal to f_s (Fig. 1). Beyond, friction decreases until a distance s_p is attained, and remains approximately constant and equal to f_k: this is a *plastic behavior*. This decrease may occur with an increase of the speed $|\dot{u}|$ (Stribeck effect), leading to stick-slip.

3. Differential models for dry friction

We model the map $u \mapsto F(u)$ by the following ODEs describing dissipative hysteresis operators ([4,5] for details):

$$\dot{x} = |\dot{u}| \cdot Ax + B\dot{u}, \quad F(u)(t) = Cx(t) + D\mathrm{sgn}\dot{u}, \quad x(0) = 0, \quad x \in \mathbb{R}^n \tag{2}$$

• A 1st order model ($n = 1$) verifies all the requirements, except the overshoot in the transient. This very simple model is a regularization of Coulomb model. It is a particularization of Dahl model [7]. • A 2nd order model has all the properties required above. They are given respectively by

$$A = -\frac{1}{\varepsilon}, \; B = \frac{f_1}{\varepsilon}, \; C = 1, \; D = 0, \quad \text{and} \quad A = -\frac{1}{\varepsilon}\begin{pmatrix} \frac{1}{\eta} & 0 \\ 0 & 1 \end{pmatrix}, \; B = \frac{1}{\varepsilon}\begin{pmatrix} \frac{f_1}{\eta} \\ -f_2 \end{pmatrix}, \; C = \begin{pmatrix} 1 & 1 \end{pmatrix}, \; D = 0$$

where $f_1 > 0$, $f_2 \geq 0$ are friction coefficients, $\varepsilon > 0$ is a distance and $\eta > 0$ is adimensionless. Changing the time variable t into the distance s defined by $ds = |\dot{u}|dt$, u (resp. $x, F(u)$) becomes u_S (resp. $x_S, F_S(u_S)$), and (2) is transformed into a *linear* differential equation:

$$\frac{dx_S}{ds} = Ax_S + B\frac{du_S}{ds}, \quad F_S(u_S) = Cx_S + D\frac{du_S}{ds}, \quad x(0) = 0$$

Remark that $\frac{du_S}{ds} = \pm 1$ according to the sense of displacement. Coulomb friction corresponds to $C = 0, D \neq 0$. Our models are linearly filtered versions of Coulomb friction. $D = 0$ is a regularization of Coulomb model, obtained when $\varepsilon \to 0$ [4,5]. In the sequel, we consider only the regularized case $D = 0$. Using s, we express the work of $F(u)$ as an integral in s:

$$\int_0^t F(u)(\tau) \cdot \dot{u}(\tau) \cdot d\tau = \int_0^{S(u)(t)} F_S(u_S) \cdot \frac{du_S}{ds}(s) \cdot ds$$

To identify parameters $f_1, f_2, \varepsilon, \eta$, we need precise definitions of physically meaningful values. We fix

$$f_k \triangleq \lim_{\substack{\dot{u}>0,\ u(t)\to+\infty, \\ t\to+\infty}} F(u)(t), \quad f_s \triangleq \sup_{\substack{u,\ t\geq 0 \\ x(0)=0}} F(u)(t) \quad \text{and} \quad k_F^{\pm} \triangleq \sup_{\substack{u,t\geq 0,\ \dot{u}(t)\neq 0 \\ x(0)=0}} \pm\frac{\dot{F}(u)(t)}{\dot{u}(t)}$$

The *kinetic friction* f_k is the asymptotical value of $F(u)$ when $\dot{u} > 0$ and s_p is a *"rise distance"*, beyond which $|F(u) - f_k|/f_k$ is less than 5%. The *static friction* f_s is the maximal value of the friction. It is attained, departing from $\dot{u} = 0$ and $x = A^{-1}B$, for $s = s_e$. s_e is the *breakaway distance from rest.* The extremal values of the slope of the hysteresis cycles u vs. F are also meaningful.

4. Principal properties of the friction models

Theorem 1 **(Hysteresis and regularity properties)**.

$F : u \mapsto F(u)$ has the hysteresis property*: $F(u \circ \varphi) = F(u) \circ \varphi$ for any increasing diffeomorphism φ on $\mathbb{R}^+$. Moreover, F is locally Lipschitz in the space of absolutely continuous functions.*

Theorem 2 **(Dissipativity condition and thermomechanical interpretation)**.

The operator $\dot{u} \mapsto F(u)$ is dissipative *if there exists a matrix P such that:*

$$P = P^T \geq 0, -A^T P - PA = Q \geq 0, C^T = PB$$

e.g. if $\varepsilon > 0$ for 1st order model and if $f_1 > f_2 \geq 0, \varepsilon > 0, 0 < \eta < 1$ for 2nd order. In this case, the power $F(u)\dot{u}$ *may be expressed, using a* pseudo-potential of dissipation $\frac{1}{2}(Qx, x)|\dot{u}|$ *and a* free energy $\frac{1}{2}(Px, x)$*, as:*

$$F(u)\dot{u} = \frac{1}{2}(Qx, x)|\dot{u}| + \frac{1}{2}\frac{d}{dt}(Px, x)$$

Theorem 3 **(Parameters identification)**.

- *The parameters values for 1st and 2nd order models are linked to the measures by*

$$\left\{ \begin{array}{l} f_k = f_1, \ \ s_p = 3\varepsilon \\ k_F^- = 0, \ \ k_F^+ = 2\frac{f_1}{\varepsilon} \end{array} \right. \quad \textit{and} \quad \left\{ \begin{array}{l} f_k = f_1 - f_2\,, \quad f_s = f_k + 2f_2\left(\frac{\eta f_2}{f_1}\right)^{\frac{\eta}{1-\eta}}(1-\eta) \\ s_e = \frac{\varepsilon\eta}{1-\eta}\log\frac{f_1}{\eta f_2}\,, \quad s_p = 3\varepsilon \\ k_F^- = \frac{2f_2}{\varepsilon}\left(\frac{\eta^2 f_2}{f_1}\right)^{\frac{\eta}{1-\eta}}(1-\eta)\,, \quad k_F^+ = \frac{2}{\eta\varepsilon}(f_1 - \eta f_2) - k_F^- \end{array} \right.$$

- *We may express the parameters as functions of measures. It is clear for 1st order. For 2nd order, define $m_1 = \frac{f_s - f_k}{f_k}$ and $m_2 = e^{3s_e/s_p}$. Suppose $f_k \geq 0$ (natural for a friction force) and $s_e < +\infty$. Then*

$$f_1 = \frac{(m_1 m_2 + 2)p}{2(p-1)}f_k, \quad f_2 = \frac{m_1 m_2 p + 2}{2(p-1)}f_k, \quad \varepsilon = \frac{s_p}{3}, \quad \eta = \frac{m_1 m_2 + 2}{m_1 m_2 p + 2}$$

where p is solution of $\frac{m_1 m_2 + 2}{m_1 m_2}\ln p = (p-1)\ln m_2$ and $p > 1$. Such a p exists and is unique iff $\ln m_2 < \frac{m_1 m_2 + 2}{m_1 m_2}$, e.g. if $3s_e < s_p$. Dissipativity is then guaranteed.

5. Application to tyre/road contact modeling

All the actions exerted on a vehicle with wheels, except gravity and aerodynamical forces, are due to tyre/road contact. This contact may be modeled [6] using a sharp physical description of the phenomena involved [17,18,11,10] or using "black box" models, Pacejka's being the most well-known [15,3,14]. First kind models lead to discretization of the contact area, second kind models give empirical formulas linking together macroscopic strains and deformations. Our models are in between: they aggregate elastic forces and Coulomb friction at each point of the contact area, and give the macroscopic forces by integral formula along the contact patch.

We consider here only the longitudinal force F_x (x, y, z, are the longitudinal, lateral and vertical (fixed) directions, ξ and ζ are linked to the tyre, cf. Fig. 2, 3). Experimentally, for a purely longitudinal motion, F_x is function of the verticale force F_z and of the slip-ratio $S_x = (V_x + \rho\dot\theta)/\rho\dot\theta$, where V_x is the vehicle longitudinal speed, ρ the tyre radius at rest and $\dot\theta$ the rotation speed of the rim around axis y. Pacejka's "Magic Formula" [3] writes here (the parameters depend upon F_z):

$$F_x = D\sin(C\arctan(B\phi)) \text{ where } \phi = (1-E)S_x + \frac{E}{B}\arctan(BS_x) \tag{3}$$

We now describe our model. We suppose that the contact patch is a rectangle of size $\ell \times w$ and that the pressure profile is uniform along ζ and ξ: $P = \frac{F_z}{w\ell}$. Model (2) furnishes the force applied to a rubber element situated in ξ at time t. Its sliding velocity is $\dot u = V_x + \rho\dot\theta$ and, $X(t,\xi)$ denoting the corresponding friction state, we have $\dot X = \frac{\partial X}{\partial t} + \rho\dot\theta\frac{\partial X}{\partial \xi}$. Moreover, we suppose that the rubber elements are at rest when entering the contact patch (by convention, we fix $\dot\theta > 0$). Finally, F_x is given by

$$\frac{\partial X}{\partial t} + \rho\dot\theta\frac{\partial X}{\partial \xi} = |\dot u|\cdot AX + B\dot u,\ \ X(0,\xi) = X(t,0) = 0,\ \ F_x(u)(t) = P\int_0^\ell CX(t,\xi)\ w\cdot d\xi \tag{4}$$

Experiments lead to take a second order model. For the steady-state deformations considered by Pacejka, ($V_x, \dot\theta$ constant), analytical formulas are obtained:

$$F_x(S_x) = -F_z\left[f_1\left(1 - \frac{\varepsilon\eta}{\ell S_x}\left(1 - e^{\frac{-\ell S_x}{\varepsilon\eta}}\right)\right) - f_2\left(1 - \frac{\varepsilon}{\ell S_x}\left(1 - e^{\frac{-\ell S_x}{\varepsilon}}\right)\right)\right]$$

which may be compared to (3) (Fig. 4, 5). Braking and traction simulations may be easily performed coupling (4) with the equations of motion for the vehicle and the tyre

$$M\dot V_x = F_x + F_{ext},\ \ J\ddot\theta = \rho F_x + T_{eng},\ \ V_x(0), \dot\theta(0) \text{ given} \tag{5}$$

where F_{ext} represents the non-frictional external forces, F_x the friction force, T_{eng} the engine torque on the wheels. This permits the simulation of stop and start, which was impossible with (3), due to the singularity at $V_x = 0$. Well-posedness of the system (4), (5) may be established using the method of characteristics and the regularity properties of the model. Main advantages on (3) are the physical meaning of the parameters, which permit extrapolation to non-stationnary case and to the lateral and self-aligning modes.

6. References

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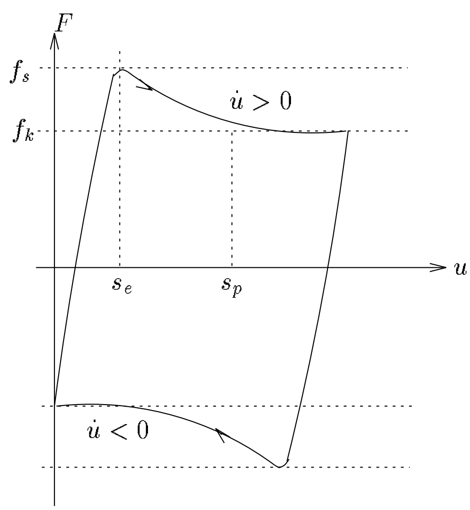

Figure 1: Hysteresis cycle with stiction

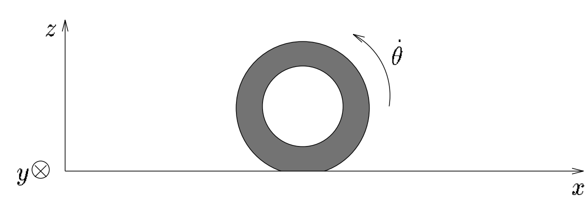

Figure 2: Tyre in the plane $x - z$

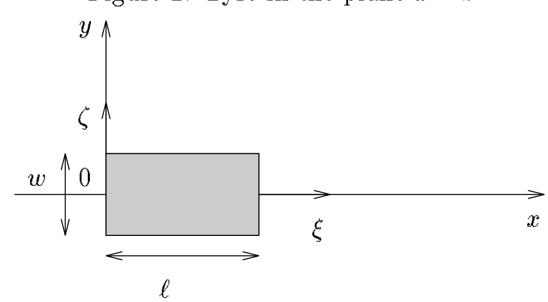

Figure 3: Tyre in the plane $x - y$

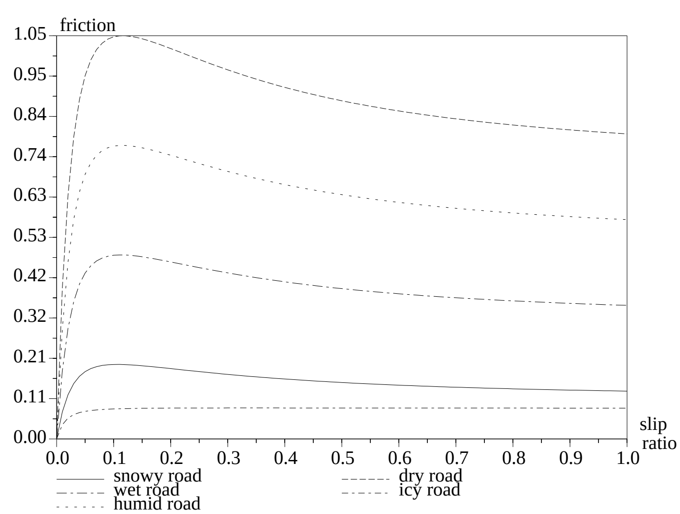

Figure 4: Adhesion curves (longitudinal behavior)

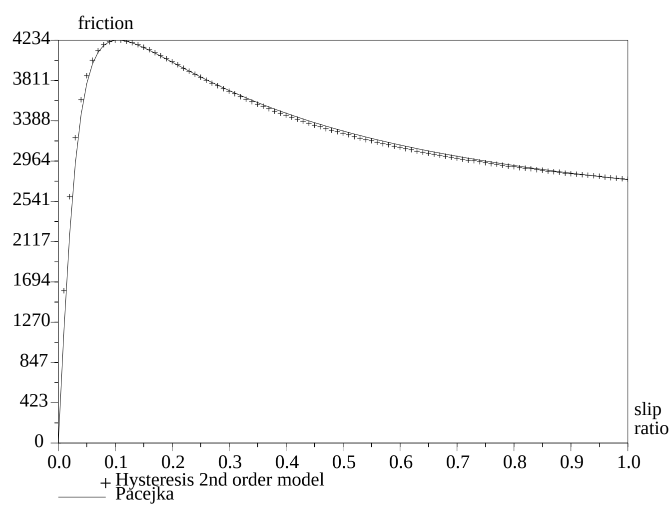

Figure 5: Comparison with Pacejka's model (longitudinal behavior)