Department of Mathematics and Constructive Geometry

Reconstruction of 3D objects from point clouds using surface evolution

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A common way of representing a 3D object is to capture a set of points lying on the object's surface – point cloud representation of an object. Point cloud data can be acquired by 3D laser scanners, RGB-D cameras (Microsoft Kinect), stereo cameras or can be artificially created by a software.

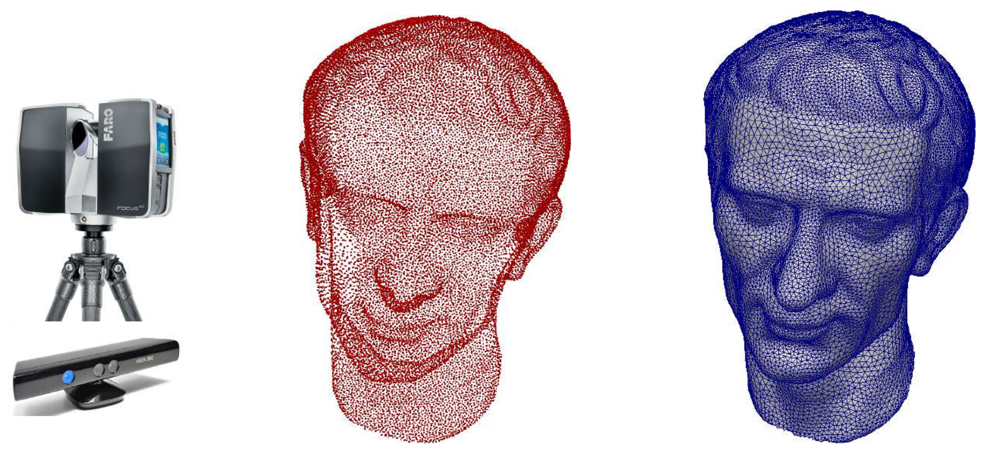


Figure 1: Point cloud representation and triangulation of a 3D object's surface.

We present a method for reconstruction of a single 3D object's surface from representative point cloud. Given this set of points, we construct a triangular mesh approximation of a surface they represent.

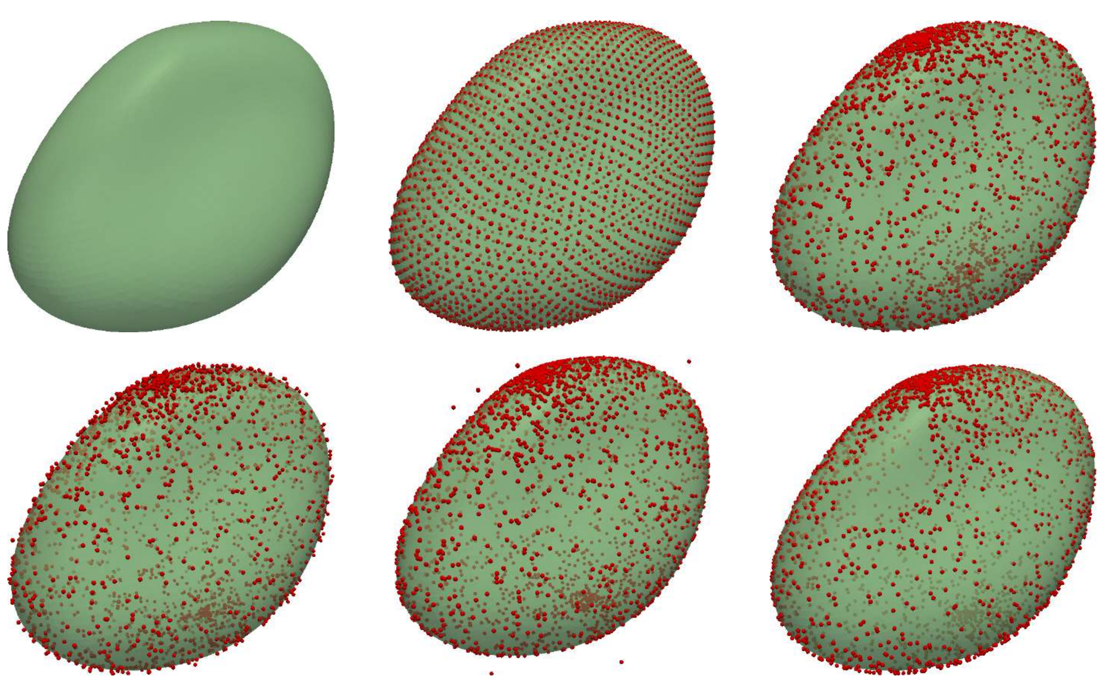


Figure 2: Different forms of point cloud representations of the testing surface.

As an input we need a set of points representing the object and an initial approximation of the desired surface. The initial approximation is a closed triangulated surface containing the given set of points in its inside. The process of surface reconstruction is based on appropriately designed evolution of this initial condition.

Surface evolution

We define *a surface evolution* (in general intrinsic setting) as any map

$$F : X \times [0, T_f] \to Y.$$

We assume F^t to be a smooth time depending embedding of 2D Riemannian manifold X in $Y \subseteq \mathbb{R}^3$, $\forall t \in [0, T_f]$. Hence:

$$\partial_t F = v = v_N + v_T$$

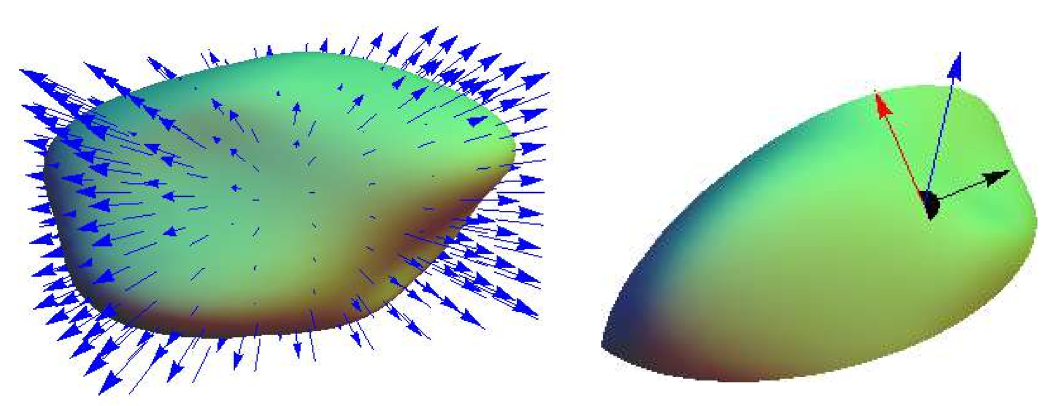


Figure 3: Decomposition of the velocity field of evolution $v = v_N + v_T$

While the movement in the normal direction directly determines the shape of the evolving surface, the tangential movement plays an important role in numerical realizations of Lagrangian evolution models. It is necessary to control the quality of the mesh during the process of evolution. Therefore, we explain the technique for the required *tangential redistribution* of the discrete points on the evolving surface.

Distance function

$$d_0 : \Omega \to \mathbb{R}, \ \Omega \subseteq \mathbb{R}^3$$

For given point cloud $P = \{p_1 \dots p_{n_P}\} \subset \Omega$ and $x \in \Omega$ we define:

$$|\nabla d_0(x)| = 1, \quad x \in \Omega, \quad d(x) = 0, \ x \in P \subset \Omega$$

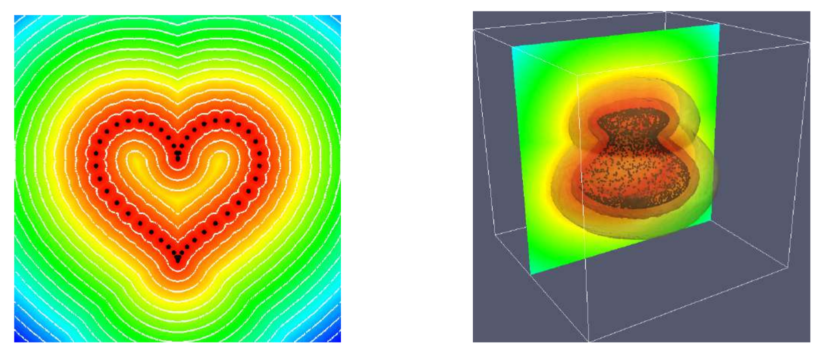


Figure 4: Examples of distance functions to the given point clouds.

Surface reconstruction model

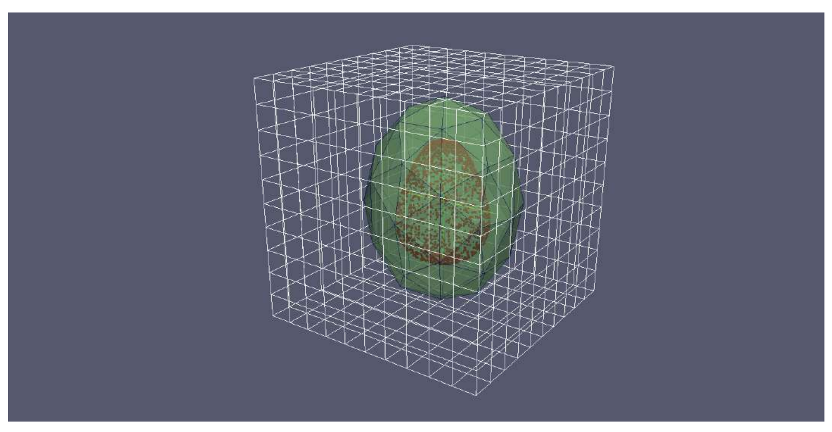
Let the point cloud P be a sampling of the surface $\Sigma \subset \mathbb{R}^3$, $d = d_0 * G_\sigma$. Let (X, g_X) be a two-dimensional Riemannian manifold without boundary and $F : X \times [0, T_f] \to \Omega$ be an evolution of X in Ω:

$$\partial_t F = w_a (-\nabla d \cdot N) N + w_d \, d \, \Delta_{g_F} F + v_T$$

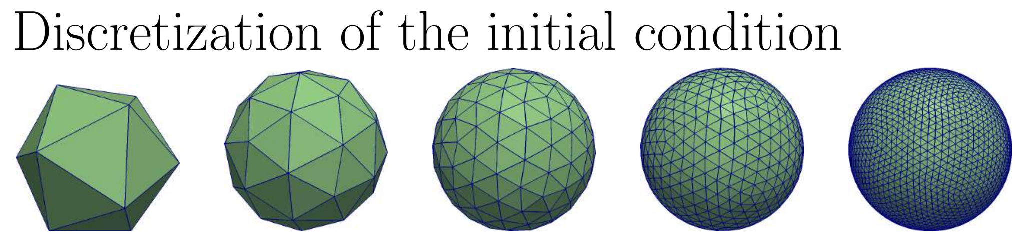
$$F(\cdot, 0) = S^0$$

Discretiazation of surface reconstruction model

- Voxelization of $\Omega \subseteq \mathbb{R}^3$ and approximation of the distance function using *Fast sweeping method*



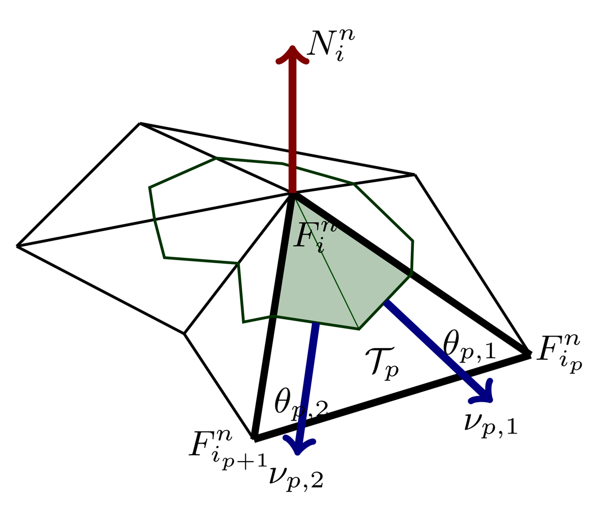
- Discretization of the initial condition

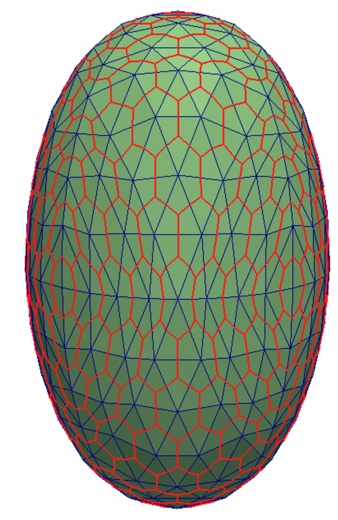


- Semi-implicit time discretization

$$\frac{F^n - F^{n-1}}{\tau} = w_a (-\nabla d \cdot N^{n-1}) N^{n-1} + w_d \, d \, \Delta_{g_{F^{n-1}}} F^n + v_T^{n-1}$$

- Space discretization





- We use the finite volume approach for approximating the proposed model

$$\int_{V_i} \frac{F^n - F^{n-1}}{\tau} \mathrm{d}\mu_{F^{n-1}} = \int_{V_i} w_a (-\nabla d \cdot N^{n-1}) N^{n-1} \mathrm{d}\mu_{F^{n-1}} + \int_{V_i} w_d \, d \, \Delta_{g_{F^{n-1}}} F^n \mathrm{d}\mu_{F^{n-1}} + \int_{V_i} v_T^{n-1} \mathrm{d}\mu_{F^{n-1}}.$$

We make use of the cotangent scheme and special version of Stokes theorem to get the fully discretized model.

Numerical experiments

We implemented the proposed method in C++ and parallelized the computations using the OpenMP API. In order to demonstrate the robustness of our method we performed numerical experiments on the testing data representing different types of the point cloud imperfections.

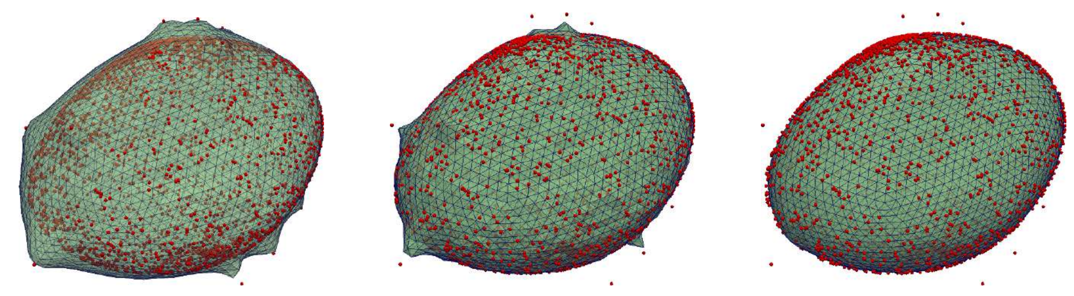


Figure 8: Reconstruction of the testing surface from a point cloud with outlying points.

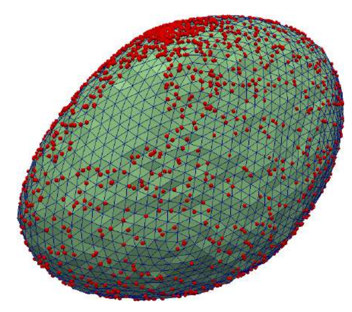


Figure 9: Reconstruction of the testing surface from a point cloud with a missing part.

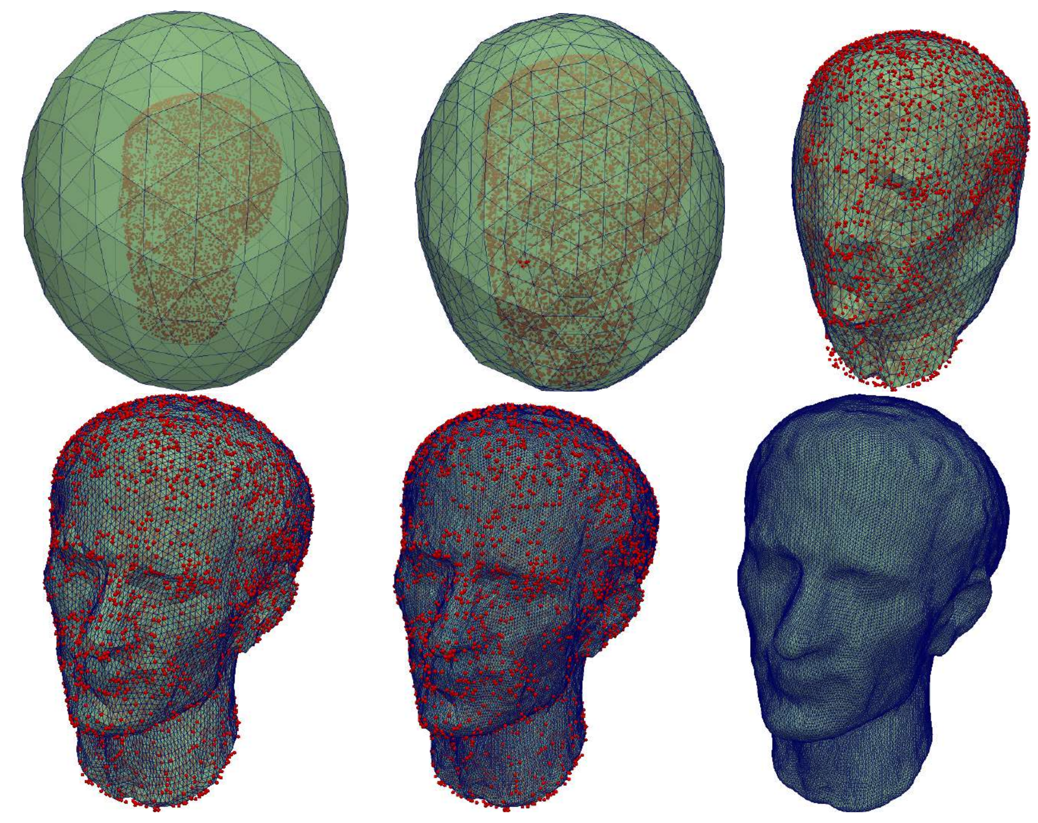


Figure 10: Reconstruction of a more complicated testing surface.

We applied the proposed surface reconstruction method to the biological data representing an early developmental stage of a zebrafish (*danio rerio*) embryo. Using our method, we are able to reconstruct surfaces from the point clouds representing forming otic vesicle (*vesicula otica*) of the embryo during the process of cell differentiation.

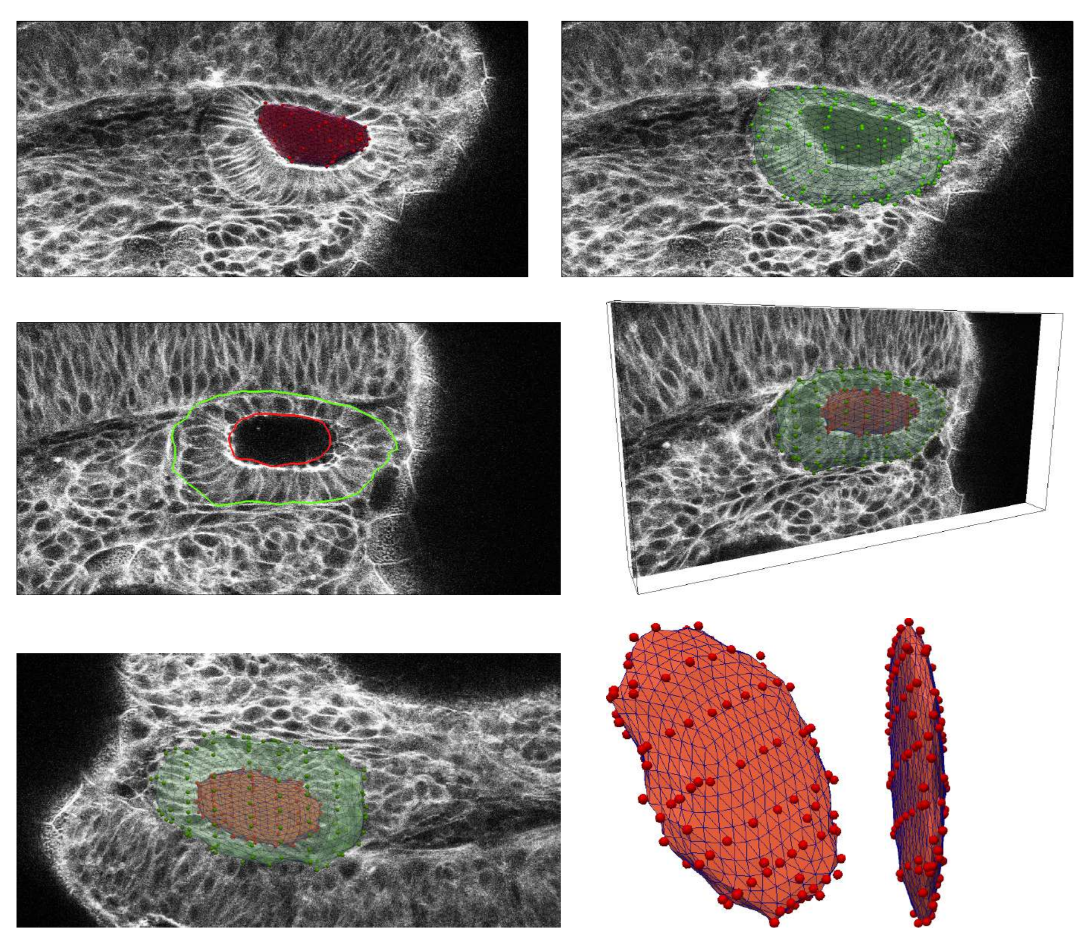


Figure 11: Reconstructed surfaces for testing embryos

The reconstructed surfaces can help in *quantitative analysis* of the embryo, e.g. determining the volume of formed structures and measuring the differences between individuals developing in different conditions.

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