Generalized Feistel Networks with Optimal Diffusion

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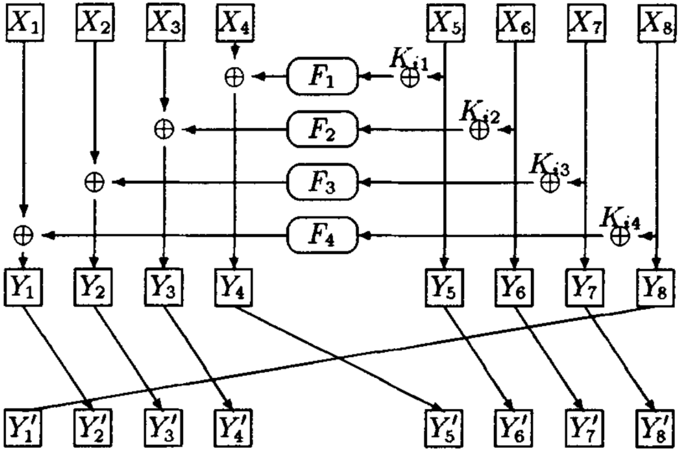
In this talk

- A new type of generalized Feistel Networks
- Linear layer design
- Wide block cipher/sponge permutation blueprint
- Fibonnaci numbers!

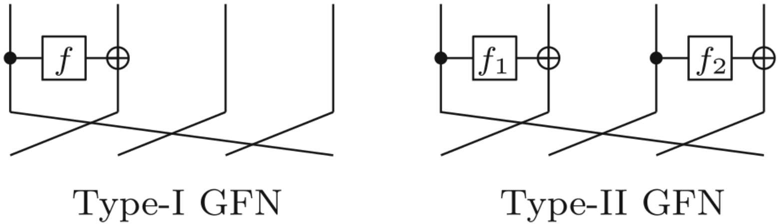
Outline

First GFN



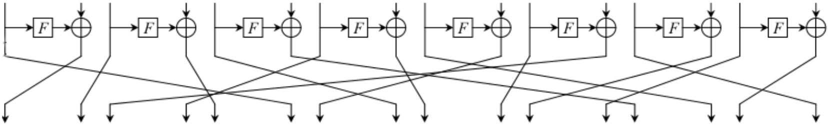
Source: *Generalized Feistel networks* , K. Nyberg (1996)

Basic GFN



Source: *Generalized Feistel networks revisited*, A. Bogdanov, K. Shibutani (2013)

Improved GFN



Source: *TWINE: A Lightweight, Versatile Block Cipher*, T. Suzaki, K. Minematsu, S. Morioka, and E. Kobayashi

Diffusion in Generalized Feistel networks

How long does it take for each input word to influence each output word?

The state consists of $2b$ branches.

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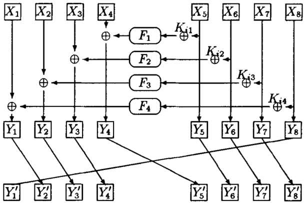

Nyberg/Type-II GFN:
$\approx 2b$ rounds

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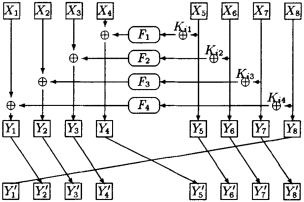
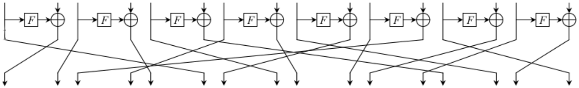
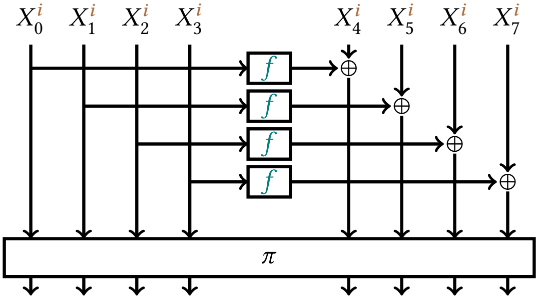




Nyberg/Type-II GFN:
$\approx 2b$ rounds

TWINE-like GFN: $\approx 2\log_2(b)$ rounds

General Vue



Optimal Diffusion

The best we can achieve is for X_0^0 to influence ϕ_{i+2} branches at round i, where

$$\phi_0 = 0, \; \phi_1 = 1, \;\; \phi_{i+2} = \phi_{i+1} + \phi_i \; .$$

Diffusion in GFNs

b	8	16	32	64	128	..	2048
Nyberg Type-II/Nyberg	16	32	64	128	256		4096
TWINE-like	6	8	10	12	14		22
Optimal	6	8	9	11	12		18

Number of rounds for full diffusion.

- Can we reach the Fibonacci-based bound?

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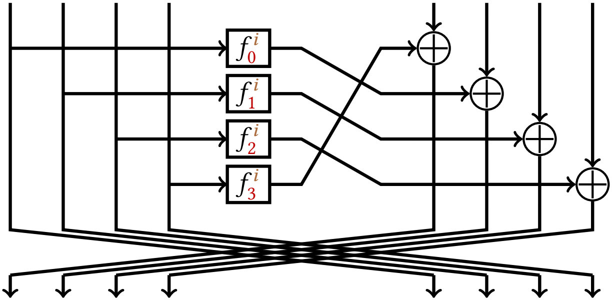
Yes (for both)

Outline

General Structure

- Number of branches: $2b$
- Number of rounds: r
- w-bit permutations f_j^i $(i < r, j < b)$
- Sequence s^i of rotations of b words.

The round i of a MRFN with $b = 4$ and $s^i = 1$ is:



Some Observations

- Both a Feistel network and a GFN
- π is very simple (1 word-wise rotation per round)
- Round function depends on the round index.
- Interesting case: $s^i = \phi_i$.

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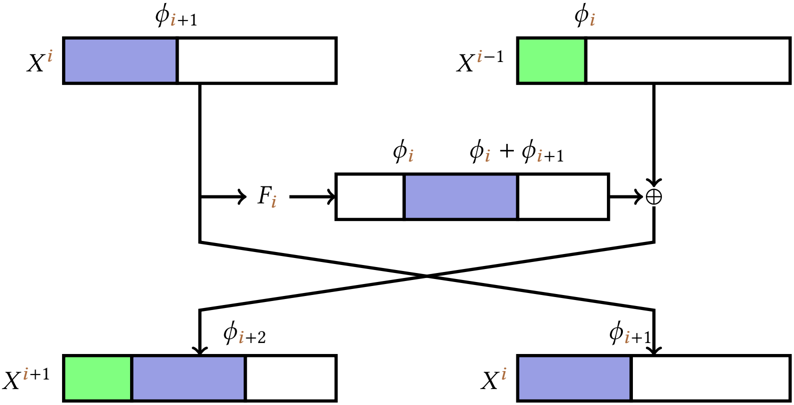
Fibonacci Case

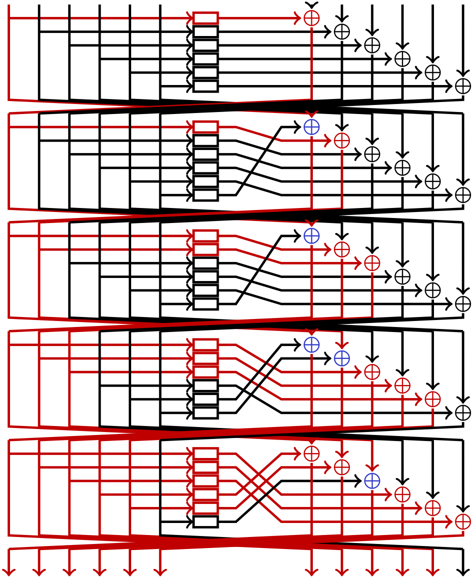
A MRFN with $s^i = \phi_i$ has optimal diffusion.

Fibonacci Case

At round 0, X_0^0 has touched the first $\phi_1 = 1$ branches of one side.



Example with 12 branches



$\phi_0 = 0$

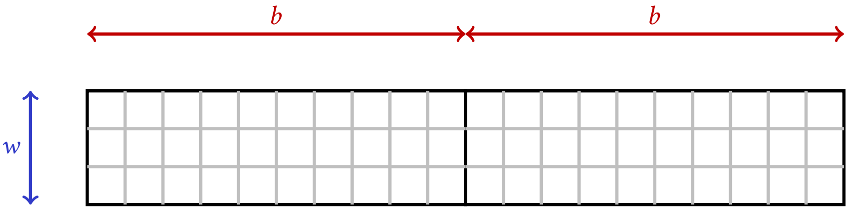
$\phi_1 = 1$

$\phi_2 = 1$

$\phi_3 = 2$

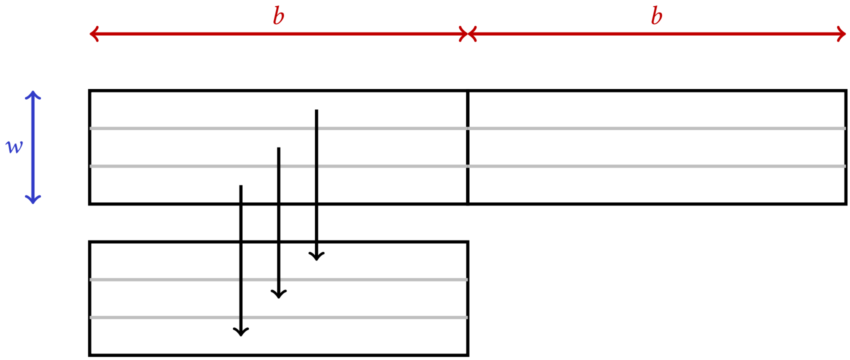
$\phi_4 = 3$

Implementation



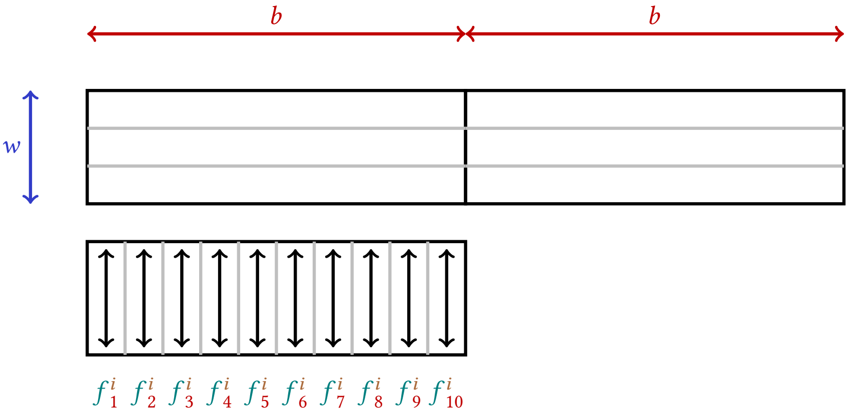
VRound function operating on $2bw$ bit internal state.

Implementation



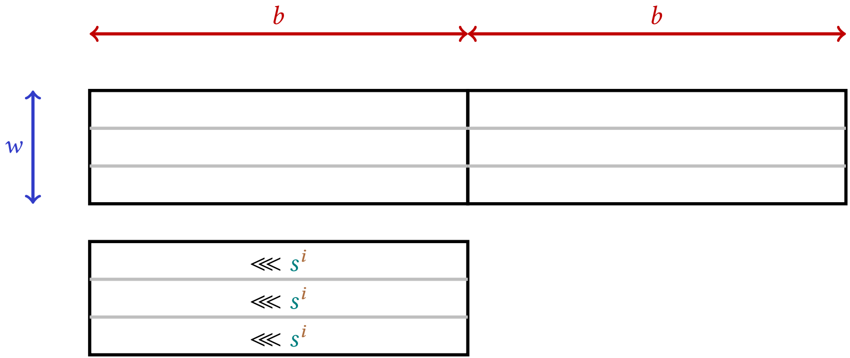
1. copy

Implementation



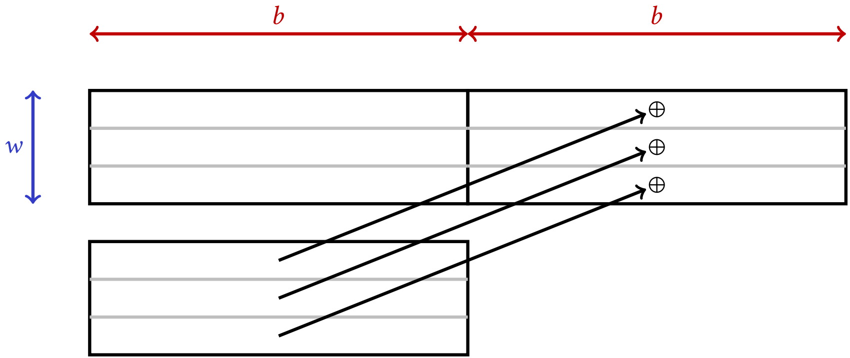
2. parallel layer of f^i

Implementation



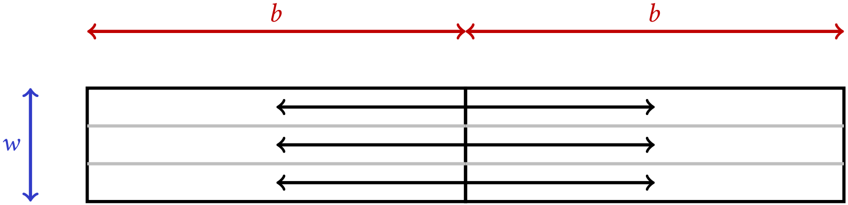
3. rotations

Implementation



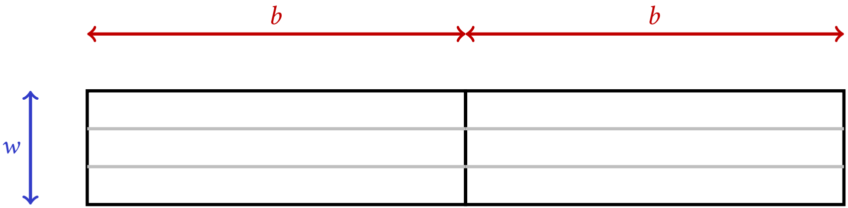
4. XOR

Implementation



5. swap

Implementation



6. finished!

Some Observations

- s^i and $s^i + (-\ell)^i \mod b$ are equivalent
- if $\gcd(s^i, b) \neq 1$ for all i, **no full diffusion!**
- Importance of the choice of $\{s^i\}_{i \geq 0}$

Security

- If $s^i = \phi_i$, then full diffusion in $\approx \Lambda(n)$ rounds, where $\Lambda(x) = i$ if $\phi_{i-1} < x \leq \phi_i$ (optimal).
- If $s^{2i} = 0$ and $i_{2i+1} = 2^i$, then full diffusion in $\approx 2\log_2(n)$ rounds (like TWINE).
- Both are quickly safe from miss-in-the-middle based impossible differential attacks and MitM!

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Open Problem 1

Differential/Linear bound?

Open Problem 2

Choice of $\{s^i\}_{i \geq 0}$?

Outline

GFN-based Linear Layers

- Use linear $\{f^i\}_{i \geq 0}$; $s^i = \phi_i$
- n-bit block divided into $2b$ branches of w bits uses:

$$\underbrace{\underbrace{\frac{w^2}{2}}_{f^i_j} \times b}_{f\,\text{layer}} \times \underbrace{2\log_2(b)}_{r} \text{ XORs}\,.$$

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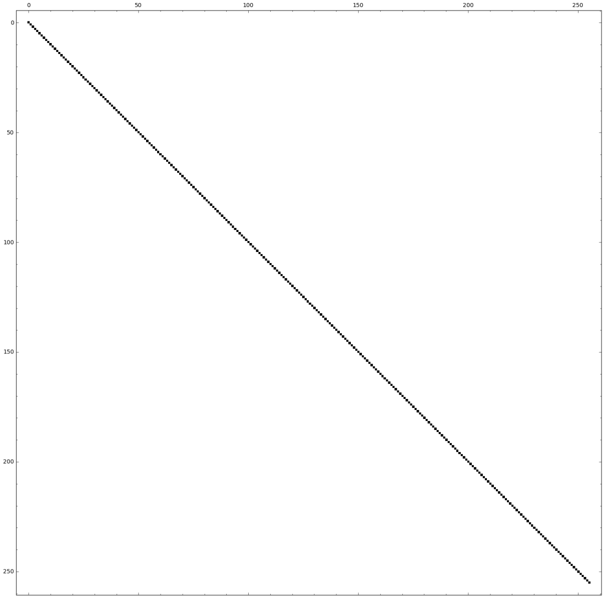
- If we fix w to a small value, then the number of XORs scales with $n\log_2(n)$ rather than n^2.
- Practical gains even for $n = 256$:

  Improvements to the Linear Layer of LowMC: A Faster Picnic, with Angela Promitzer, Sebastian Ramacher and Christian Rechberger (2017/448)

Example of Linear Layer

- $n = 256$
- $w = 4$
- $b = 32$

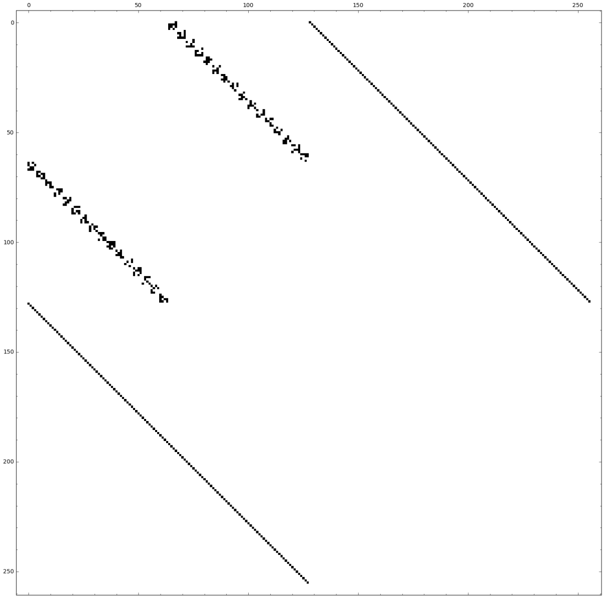
$i = 0$



Example of Linear Layer

- $n = 256$
- $w = 4$
- $b = 32$

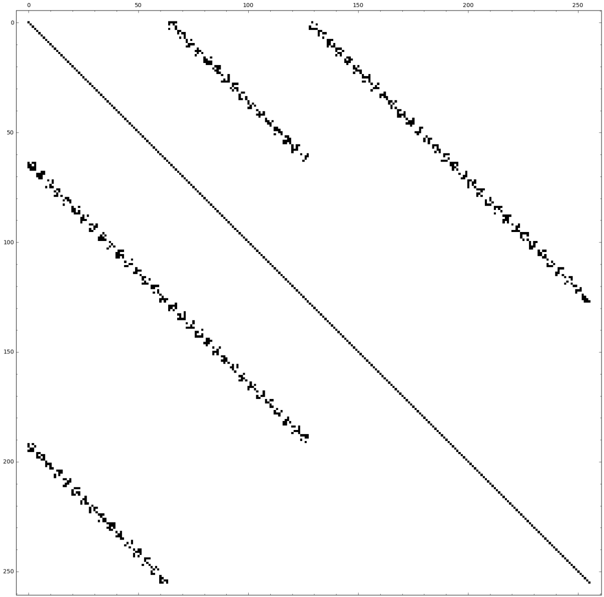
$i = 1$



Example of Linear Layer

- $n = 256$
- $w = 4$
- $b = 32$

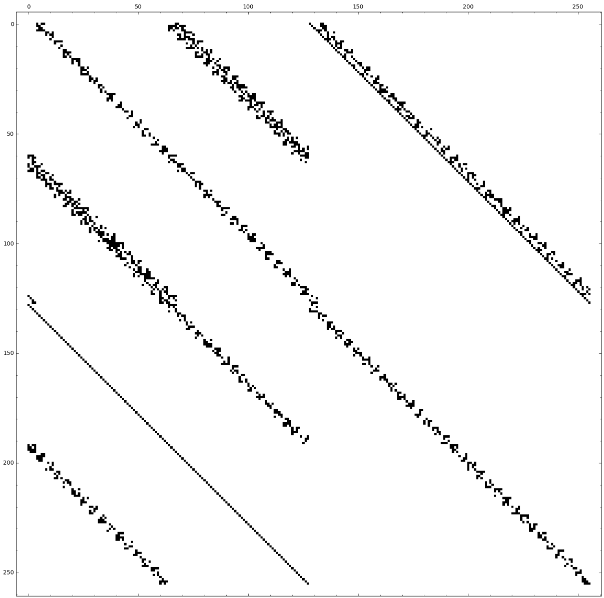
$i = 2$



Example of Linear Layer

- $n = 256$
- $w = 4$
- $b = 32$

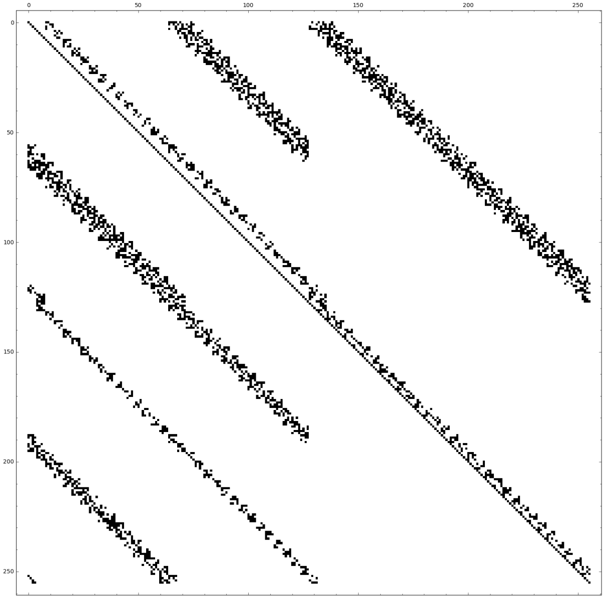
$i = 3$



Example of Linear Layer

- $n = 256$
- $w = 4$
- $b = 32$

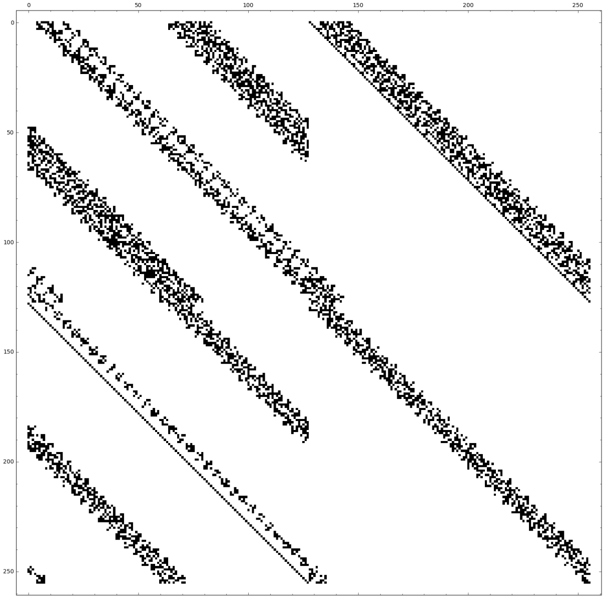
$i = 4$



Example of Linear Layer

- $n = 256$
- $w = 4$
- $b = 32$

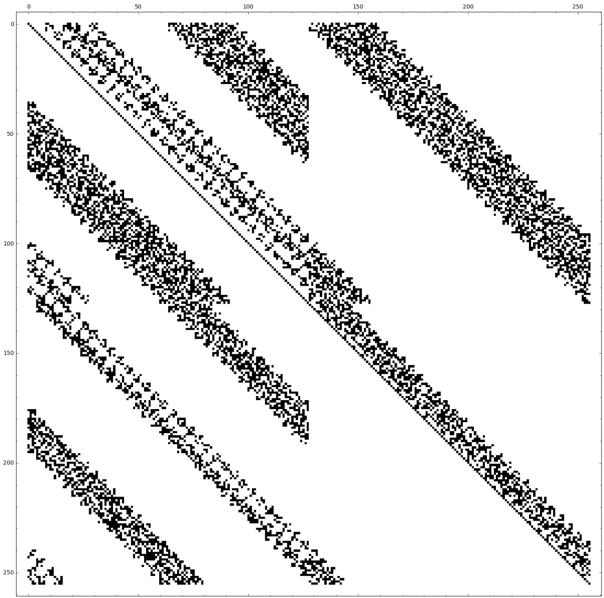
$i = 5$



Example of Linear Layer

- $n = 256$
- $w = 4$
- $b = 32$

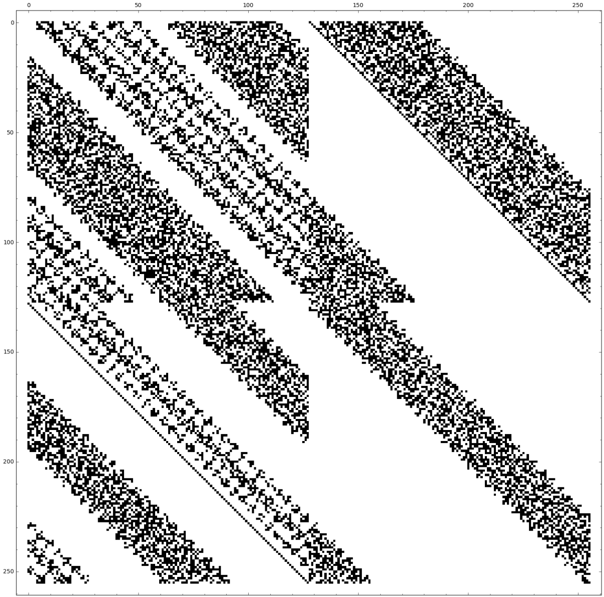
$i = 6$



Example of Linear Layer

- $n = 256$
- $w = 4$
- $b = 32$

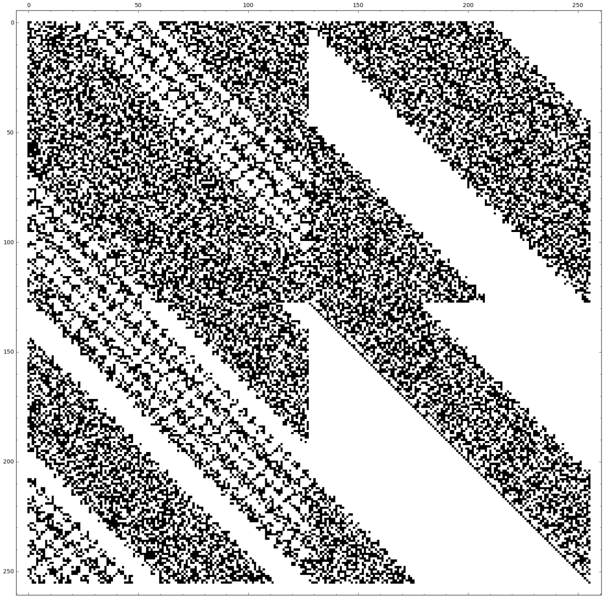
$i = 7$



Example of Linear Layer

- $n = 256$
- $w = 4$
- $b = 32$

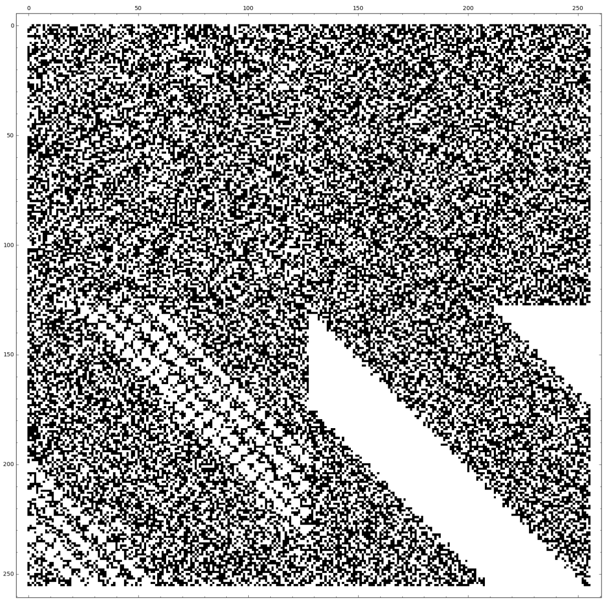
$i = 8$



Example of Linear Layer

- $n = 256$
- $w = 4$
- $b = 32$

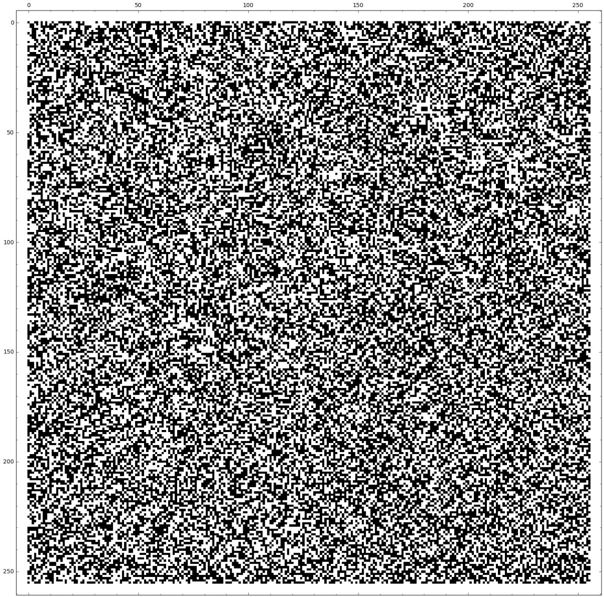
$i = 9$



Example of Linear Layer

- $n = 256$
- $w = 4$
- $b = 32$

$i = 10$



Sponge function?

- $n = 384$, with $b = 64$ and $w = 3$
- $f_j^i(x) = \chi_3(x \oplus c_j^i)$
- $s^{2i} = 0, s^{2i+1} = 2^i$ for $0 \leq i < 2\log_2(b) = 12$, then repeat (4? times):

$$s = \{0, 1, 0, 2, 0, 4, 0, 8, 0, 16, 0, 32\}$$

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Efficiency estimates

On 64-bit processors, for each round:

- 3 word copies
- 3 word-wise AND
- 3+3+3 word-wise XORs
- **Maybe** safe for 48 rounds if ≥ 8 active f functions/round on average.

Other?

- MiMC-like construction where $f^i_j(x) = (x + c^i_j)^3$ (what Arnab just presented).

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- You tell me!

Outline

Conclusion

Fun stuff happens when we allow the use of different permutations in each round!

Open problems

1. What are good sequences of rotations?
2. How to bound number of active f functions?
3. What can we use it for?
4. What happens in other structures (SPN? ARX?) when the linear layers are round-dependent?

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Thank you!