

Spectrum-Revealing Matrix Factorizations Theory and Algorithms

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Joint work with D. Anderson, J. Deursch, C. Melgaard, J. Xiao

Content

- ▶ **Low-rank matrix approximations**
- ▶ Rank-revealing matrix factorizations vs. Spectrum-revealing matrix factorizations.
- ▶ Related new algorithms
- ▶ Numerical Experiments
- ▶ Future work

Background–Applications: Bing Search

- ▶ Scientific Computing/Data Analysis
- ▶ Computer Vision
- ▶ Machine learning; kernel learning.

The screenshot shows a Bing search results page for the query "low-rank approximation". The search bar at the top contains the text "low-rank approximation" and a magnifying glass icon. Below the search bar are navigation tabs for "Web", "Images", "Videos", "Maps", "News", and "Explore", with "Images" selected. The page features a "Low Rank Badge" and a "Low Rank Score". Navigation filters include "Image size", "Color", "Type", "Layout", "People", "Date", "License", and "SafeSearch: Moderate".

The search results include several items:

- Comparison of the methods:** A graph comparing the performance of several methods: "Exact SVD", "Ranking SVD", "Infection-based", and "Proposed method". The x-axis is labeled "Time (sec.)" with values 10^1 , 10^2 , 10^3 , and 10^4 . The y-axis is labeled 10^0 , 10^1 , and 10^2 . A text box below the graph states: "Structured low-rank approximation for system identification: an example from DASY database. It is 1015 block-Hankel matrix with 2 x 3 blocks (example #13 from the abstract)." Below the graph is the text "© 2012".
- Our Technique:** A slide titled "Our Technique" with a list of bullet points:
 - Better subspace embedding!
 - Define $k \times n$ matrix S , for $k = \text{poly}(d)$
 - S is really sparse: single randomly chosen non-zero entry per columnBelow the text is a matrix representation:
$$\begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
- Low rank approximation:** A slide titled "Low rank approximation" with the following text:
 - Rank matrix: $m \times n$ where n is the sample size
 - Large n causes computational problems
 - e.g. Inversion, eigenvalue computation costs $O(n^3)$ in time.
 - Low rank approximation
 - $X \approx UV^T$, where $U, V \in \mathbb{R}^{m \times r}$ matrix ($r < n$)
 - The decay of eigenvalues of a rank matrix is often quite fast (low-rank SVD, rank-trace is sparse entry).
- Nonlocal Sparse and Low-Rank Regularization for Optical Flow Estimation:** A slide from IIS (Institute of Information Systems) featuring the text "Nonlocal Sparse and Low-Rank Regularization for Optical Flow Estimation" and "IIS TECHNOLOGIES". It includes contact information for IIS at the University of Bonn.
- Low Rank Approximation:** A slide titled "Low Rank Approximation" with the subtitle "Optimal, Sparse-based Applications".
- Robust Subspace Estimation Using Low-Rank Optimization:** A slide titled "Robust Subspace Estimation Using Low-Rank Optimization" with the subtitle "Theory and Applications".

Google Search on Low-rank Approximation

Google neural network matrix completion MRI nuclear norm

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SafeSearch Settings

Figure 1: Matrix completion and reconstruction of a large matrix. The figure shows a grid of images reconstructed from a low-rank approximation of a matrix. The images are arranged in a grid, with the top row showing the original images and the bottom row showing the reconstructed images. The reconstruction quality is compared for different values of the rank r and the regularization parameter λ .

Figure 2: Matrix completion. The figure shows a grid of images reconstructed from a low-rank approximation of a matrix. The images are arranged in a grid, with the top row showing the original images and the bottom row showing the reconstructed images. The reconstruction quality is compared for different values of the rank r and the regularization parameter λ .

Figure 3: Matrix completion. The figure shows a grid of images reconstructed from a low-rank approximation of a matrix. The images are arranged in a grid, with the top row showing the original images and the bottom row showing the reconstructed images. The reconstruction quality is compared for different values of the rank r and the regularization parameter λ .

Figure 4: Matrix completion. The figure shows a grid of images reconstructed from a low-rank approximation of a matrix. The images are arranged in a grid, with the top row showing the original images and the bottom row showing the reconstructed images. The reconstruction quality is compared for different values of the rank r and the regularization parameter λ .

Figure 5: Matrix completion. The figure shows a grid of images reconstructed from a low-rank approximation of a matrix. The images are arranged in a grid, with the top row showing the original images and the bottom row showing the reconstructed images. The reconstruction quality is compared for different values of the rank r and the regularization parameter λ .

Figure 6: Matrix completion. The figure shows a grid of images reconstructed from a low-rank approximation of a matrix. The images are arranged in a grid, with the top row showing the original images and the bottom row showing the reconstructed images. The reconstruction quality is compared for different values of the rank r and the regularization parameter λ .

Figure 7: Matrix completion. The figure shows a grid of images reconstructed from a low-rank approximation of a matrix. The images are arranged in a grid, with the top row showing the original images and the bottom row showing the reconstructed images. The reconstruction quality is compared for different values of the rank r and the regularization parameter λ .

Figure 8: Matrix completion. The figure shows a grid of images reconstructed from a low-rank approximation of a matrix. The images are arranged in a grid, with the top row showing the original images and the bottom row showing the reconstructed images. The reconstruction quality is compared for different values of the rank r and the regularization parameter λ .

Figure 9: Matrix completion. The figure shows a grid of images reconstructed from a low-rank approximation of a matrix. The images are arranged in a grid, with the top row showing the original images and the bottom row showing the reconstructed images. The reconstruction quality is compared for different values of the rank r and the regularization parameter λ .

Figure 10: Matrix completion. The figure shows a grid of images reconstructed from a low-rank approximation of a matrix. The images are arranged in a grid, with the top row showing the original images and the bottom row showing the reconstructed images. The reconstruction quality is compared for different values of the rank r and the regularization parameter λ .

Goals in Low-rank Matrix Approximations

- ▶ Highly efficient
- ▶ Minimum communication
- ▶ As accurate/reliable as truncated SVD
- ▶ Beyond SVD: preserve sparsity, structure, interpretability

Background: Low-Rank Approximation Methods

- ▶ Deterministic low-rank approximations
 - ▶ Interpolative decomposition (ID) (Cheng et. al. 2005, Liberty et. al 2007)
 - ▶ Deterministic CUR with various column selection algorithms
- ▶ Randomized low-rank approximations
 - ▶ Using the subsampled fft technique, for $\ell \gg k$: "optimal" complexity is
$$O(mn \log(k) + \ell^2(m + n))$$
 - ▶ Several approximation algorithms (Halko et. al. 2010)
 - ▶ Some of the first randomized algorithms. (Rokhlin et. al. 2008)
 - ▶ Randomized LU decomposition for low-rank approximation (Shabat et. al. 2015)

Spectrum-revealing factorizations stronger in theory and efficiency

Background: Other Data Analysis Methods

Data Analysis: Low-rank matrix approximations that respect data sparsity, structure, and interpretability; more informative than truncated SVD.

- ▶ CX decomposition: for medical data, text data, etc

$$A \approx CX,$$

where C consists of well-chosen columns of A .

- ▶ CUR decomposition: for recommendation systems, text data analysis

$$A \approx CUR,$$

where C and R consist of well-chosen columns and rows of A .

- ▶ LHL^T decomposition: Approximate kernel methods, independent component analysis

$$A \approx LHL^T,$$

where L consists of well-chosen columns of SPD matrix A .

Google Search on CUR Decomposition



cur decomposition genetics

Web

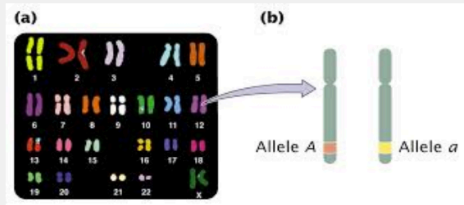
Images

News

Videos

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Golden Standard: Truncated SVD

Given matrix $M \in \mathbf{R}^{m \times n}$ with $m \leq n$, its SVD is

$$M = U\Sigma V^T = \begin{pmatrix} u_1 & \cdots & u_n \end{pmatrix} \begin{pmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_n \end{pmatrix} \begin{pmatrix} v_1 & \cdots & v_n \end{pmatrix}^T,$$

where σ_j is the j^{th} largest singular value of M . The rank- k truncated SVD is

$$M_k = U_k \Sigma_k V_k^T = \begin{pmatrix} u_1 & \cdots & u_k \end{pmatrix} \begin{pmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_k \end{pmatrix} \begin{pmatrix} v_1 & \cdots & v_k \end{pmatrix}^T.$$

Theorem: (Eckart & Young, 1936)

$$\mathbf{Min}_{\text{rank}(H) \leq k} \|M - H\|_2 = \|M - M_k\|_2 = \sigma_{k+1}(M).$$

Review: Rank-revealing Matrix factorizations: QR

- ▶ **Given:** Matrix $M \in \mathbb{C}^{m \times n}$ and $1 \leq \ell \leq \min(m, n) = n$,
Existence: QR factorization with column permutation Π

$$M\Pi = Q \begin{pmatrix} A_\ell & B_\ell \\ 0 & C_\ell \end{pmatrix} \quad \text{with}$$

$$\frac{\sigma_j(M)}{\sqrt{1 + \ell(n - \ell)}} \leq \sigma_j(A_\ell) \leq \sigma_j(M), \quad j = 1, \dots, \ell,$$
$$\sigma_{\ell+1}(M) \leq \|C_\ell\|_2 \leq \sigma_{\ell+1}(M) \sqrt{1 + \ell(n - \ell)},$$

where $\sigma_j(\cdot)$ is the j^{th} largest singular value.

Rank-revealing: A_ℓ reveals numerical rank of M .

Example: Kahan Matrix

- ▶ For $s, c > 0$ and $s^2 + c^2 \leq 1$, Kahan Matrix $K_n = S_n C_n$:
$$S_n = \mathbf{diag}(1, s, \dots, s^{n-1}), C_n = \begin{pmatrix} 1 & -c & -c & \cdots & -c \\ & 1 & -c & \cdots & -c \\ & & 1 & \cdots & -c \\ & & & \ddots & \vdots \\ & & & & 1 \end{pmatrix}$$

- ▶ Kahan Matrix not in rank revealed form:

$$\sigma_{\min}(K_n) = O\left(\left(\frac{s}{1+c}\right)^n\right) \ll s^{n-1}$$

- ▶ Little consensus on definition of "reveal."

New: Spectrum-revealing factorizations (I): QR/CX

- ▶ **Given:** Matrix $M \in \mathbb{C}^{m \times n}$ and $1 \leq k \leq \ell \leq \min(m, n) = n$,
Existence: QR factorization with column permutation Π

$$M\Pi = Q \begin{pmatrix} A_\ell & B_\ell \\ 0 & C_\ell \end{pmatrix} \quad \text{with} \quad \tau_j = \frac{\sigma_{\ell+1}(M)}{\sigma_j(M)} \quad j = 1, \dots, k+1,$$

$$\frac{\sigma_j(M)}{\sqrt{1 + O(\tau_j^2)}} \leq \sigma_j \left(\begin{pmatrix} A_\ell & B_\ell \end{pmatrix} \right) \leq \sigma_j(M)$$

$$\sigma_{k+1}(M) \leq \|M\Pi - QM_k\| \leq \sigma_{k+1}(M) \sqrt{1 + O(\tau_{k+1}^2)}$$

$$\left(\text{cf. } \frac{\sigma_j(M)}{\sqrt{1 + \ell(n-\ell)}} \leq \sigma_j(A_\ell), \quad \|C_\ell\|_2 \leq \sigma_{\ell+1}(M) \sqrt{1 + \ell(n-\ell)}. \right)$$

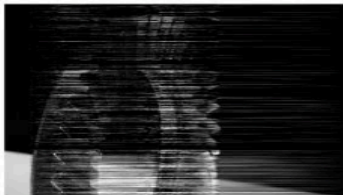
$M_k =$ truncated SVD of $\begin{pmatrix} A_\ell & B_\ell \\ 0 & 0 \end{pmatrix}$; $\sigma_j(\cdot) = j^{\text{th}}$ largest singular value.

QM_k well approximates $M\Pi$ and reveals its leading singular values.

Original $k=2442$



Truncated QR $k=244$



Truncated QRCP $k=244$



Truncated RQRCP $k=244$



New: Spectrum-revealing factorizations (II) Cholesky

- ▶ **Given:** SPD matrix $M \in \mathbb{R}^{n \times n}$ and $1 \leq k \leq \ell \leq n$,
Existence: Cholesky factorization with diagonal permutation

$$\Pi M \Pi^T = LL^T, \quad L = \begin{pmatrix} A_\ell & 0 \\ B_\ell & C_\ell \end{pmatrix} \quad \text{with} \quad \tau_j = \frac{\sigma_{\ell+1}(M)}{\sigma_j(M)}$$

$$\frac{\sigma_j(M)}{1 + O(\tau_j)} \leq \sigma_j^2 \begin{pmatrix} A_\ell \\ B_\ell \end{pmatrix} \leq \sigma_j(M), \quad j = 1, \dots, k,$$

$$\sigma_{k+1}(M) \leq \|\Pi M \Pi^T - L_k L_k^T\| \leq \sigma_{k+1}(M) (1 + O(\tau_{k+1}))$$

$$L_k = \text{truncated SVD of } \begin{pmatrix} A_\ell \\ B_\ell \end{pmatrix}.$$

New: Spectrum-revealing factorizations (II) LHL^T

- ▶ **Given:** SPD matrix $M \in \mathbb{R}^{n \times n}$ and $1 \leq k \leq \ell \leq n$,
- ▶ **Existence:** LHL^T factorization with diagonal permutation

$$\Pi M \Pi^T = LL^T, \quad L = \begin{pmatrix} A_\ell & 0 \\ B_\ell & C_\ell \end{pmatrix},$$

$$\hat{M} = \Pi^T \begin{pmatrix} A_\ell \\ B_\ell \end{pmatrix} \begin{pmatrix} A_\ell \\ B_\ell \end{pmatrix}^\dagger (\Pi M \Pi^T) \begin{pmatrix} A_\ell \\ B_\ell \end{pmatrix} \begin{pmatrix} A_\ell \\ B_\ell \end{pmatrix}^\dagger \Pi$$

$$\frac{\sigma_j(M)}{\sqrt{1 + O(\tau_j^2)}} \leq \sigma_j(\hat{M}) \leq \sigma_j(M), \quad j = 1, \dots, k$$

$$\sigma_{k+1}(M) \leq \|M - \hat{M}_k\| \leq \sigma_{k+1}(M) \sqrt{1 + O(\tau_{k+1}^2)}$$

$$\hat{M}_k = \text{truncated SVD of } \hat{M}; \quad \tau_j = \frac{\sigma_{\ell+1}(M)}{\sigma_j(M)}$$

New: Spectrum-revealing factorizations (III) LU

- ▶ **Given:** Matrix $M \in \mathbb{C}^{m \times n}$ and $1 \leq k \leq \ell \leq \min(m, n) = n$,
Existence: LU with row, column permutations Π_l, Π_r

$$\Pi_l M \Pi_r = LU, \quad L = \begin{pmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{pmatrix}, \quad U = \begin{pmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{pmatrix}$$

$$\text{with } \tau_j = \frac{\sigma_{\ell+1}(M)}{\sigma_j(M)}, \quad j = 1, \dots, \ell$$

$$\frac{\sigma_j(M)}{1 + O(\tau_j)} \leq \sigma_j \left(\begin{pmatrix} L_{11} \\ L_{21} \end{pmatrix} \cdot (U_{11} \quad U_{12}) \right) \leq \sigma_j(M),$$

$$\sigma_{k+1}(M) \leq \|M - \hat{M}_k\| \leq \sigma_{k+1}(M) (1 + O(\tau_{k+1}))$$

$$\hat{M}_k = \text{truncated SVD of } \begin{pmatrix} L_{11} \\ L_{21} \end{pmatrix} \cdot (U_{11} \quad U_{12}).$$

New: Spectrum-revealing factorizations (III) CUR

- **Given:** Matrix $M \in \mathbb{C}^{m \times n}$ and $1 \leq k \leq \ell \leq \min(m, n) = n$,
Existence: CUR with row, column permutations Π_l, Π_r

$$\Pi_l M \Pi_r = LU, \quad L = \begin{pmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{pmatrix}, \quad U = \begin{pmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{pmatrix}$$

$$\Pi_l \hat{M} \Pi_r = \begin{pmatrix} L_{11} \\ L_{21} \end{pmatrix} \begin{pmatrix} L_{11} \\ L_{21} \end{pmatrix}^\dagger (\Pi_l M \Pi_r) \begin{pmatrix} U_{11} & U_{12} \end{pmatrix}^\dagger \begin{pmatrix} U_{11} & U_{12} \end{pmatrix}.$$

$$\frac{\sigma_j(M)}{\sqrt{1 + O(\tau_j^2)}} \leq \sigma_j(\hat{M}) \leq \sigma_j(M), \quad j = 1, \dots, k$$

$$\sigma_{k+1}(M) \leq \|M - \hat{M}_k\| \leq \sigma_{k+1}(M) \sqrt{1 + O(\tau_{k+1}^2)}$$

$$\hat{M}_k = \text{truncated SVD of } \hat{M}; \quad \tau_j = \frac{\sigma_{\ell+1}(M)}{\sigma_j(M)}$$

Talks: Spectrum-Revealing Matrix Factorizations and Randomized Algorithms

True BLAS-3 QRCP with Random Sampling and Low Rank Approximations

Jed Duersch Ming Gu

UC Berkeley, Mathematics

November 25, 2015

Jed Duersch, Ming Gu

1/43

Spectrum-Revealing truncated Cholesky Theory and Algorithms

Jianwei Xiao

Department of Mathematics
University of California, Berkeley

December, 9, 2015
Joint work with Ming Gu

Navigation icons

Efficient Robust Randomized GECP (Gaussian Elimination with Complete Pivoting)

Christopher Melgaard

Joint work with Ming Gu.

Navigation icons

An Efficient Randomized Algorithm for Computing a Spectrum-Revealing LU Decomposition

David G. Anderson

Department of Mathematics,
University of California, Berkeley
Joint work with Ming Gu

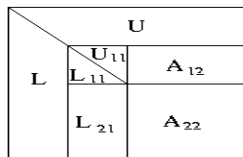
April 29, 2015

New Algorithms (1a)

- ▶ Solving linear systems of equations $Ax = b$:
 - ▶ Standard method: Gaussian Elimination with Partial Pivoting (GEPP): $\Pi A = LU$: most efficient, but can be unstable.
 - ▶ New method: randomized Gaussian Elimination with Complete Pivoting (RGECP): $\Pi_l A \Pi_r = LU$: almost as efficient, but is always stable. (joint work with C. Melgaard.)
 - ▶ Communication-avoiding version would be numerically stable, in contrast to CA-LU with partial pivoting.

New Algorithms (Ib)

- ▶ Column pivots random: **Successive Schur Sampling**.
- ▶ RGECP is $O(rn^2)$ more flops than GEPP:



- ▶ **Theorem:** Given $\delta \in (0, 1)$. If

$$r > 32 \ln \left(\frac{n(n+1)}{2\delta} \right),$$

then the pivot growth factor of RGECP satisfies

$$\rho(A) \leq 3\sqrt{e(n+1)}n^{2+\frac{1}{2}\ln(n)}$$

with probability greater than $1 - \delta$.

New Algorithms (IIa)

- ▶ Low-rank Matrix Approximations

- ▶ Randomized algorithm for computing a Spectrum-revealing LU Decomposition (joint work with D. Anderson): First of its kind. Computes

$$\Pi_l A \Pi_r \approx LU, \quad L = \begin{pmatrix} L_{11} \\ L_{21} \end{pmatrix}, \quad U = \begin{pmatrix} U_{11} & U_{12} \end{pmatrix}$$

without ever computing the Schur complement. Likely the most efficient for low-rank matrix approximation.

- ▶ Non-classical communication patterns:
 - ▶ Computing/Updating random projection.
 - ▶ Both row and column swaps.
 - ▶ No Schur updating.

New Algorithms (IIb)

MAIN ALGORITHM: Truncated Randomized LUCP

Input: matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$, target rank k , oversampling parameter p

Output: $\mathbf{L} \in \mathbb{R}^{m \times k}$, $\mathbf{U} \in \mathbb{R}^{k \times n}$, with $\mathbf{LU} \approx \mathbf{A}$

b	$<$	p	$<$	k	\leq	ℓ	\ll	m, n
block size		our oversamp. param.		target rank		prev. work oversamp. param		dim.

Initialization: compress matrix with random projection $\Omega \in \mathbb{R}^{p \times m}$:

$\mathbf{R} = \Omega \mathbf{A}$ Iterate:

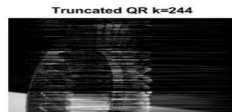
1. Perform column selection (QR or LU with partial column pivoting) on \mathbf{R} to determine column pivots
2. Update block column of \mathbf{L}
3. Perform iteration of block LU with partial row pivots on \mathbf{A}
4. Update block row of \mathbf{U}
5. Update projection so that \mathbf{R} is a random projection of the Schur complement

New Algorithms (III)

- ▶ Low-rank Matrix Approximations
 - ▶ Randomized algorithm for computing a Spectrum-revealing QR factorization (J. Duersch, Nov. 2015)

$$A\Pi \approx Q \begin{pmatrix} R_{11} & R_{12} \\ 0 & 0 \end{pmatrix}$$

Never need to update trailing submatrix in R . Up to twice as fast as **truncated QR**.

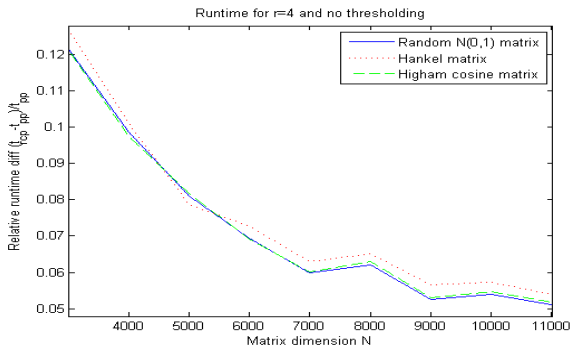


New Algorithms (IV)

- ▶ Low-rank Matrix Approximations
 - ▶ Randomized algorithm for computing a Spectrum-revealing Cholesky factorization
Randomized block left looking Cholesky. Much promise in quality and efficiency.

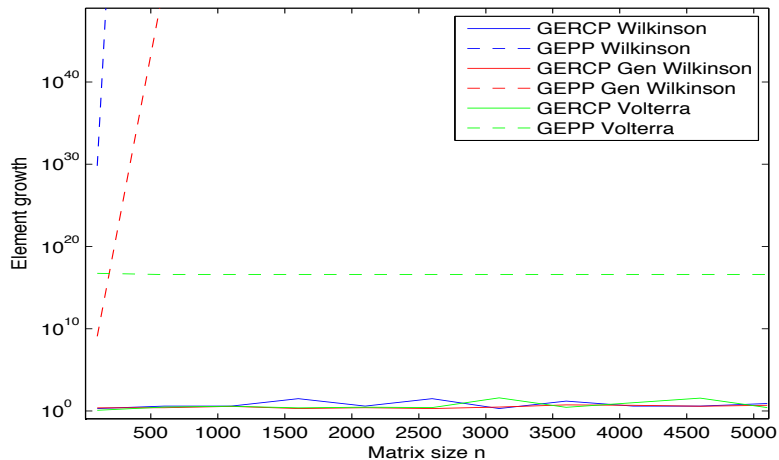
Numerical Experiments for RGECP (I)

- ▶ Fortran implementation, optimized BLAS.
- ▶ Randomized GECP: Execution Time Relative to GEPP



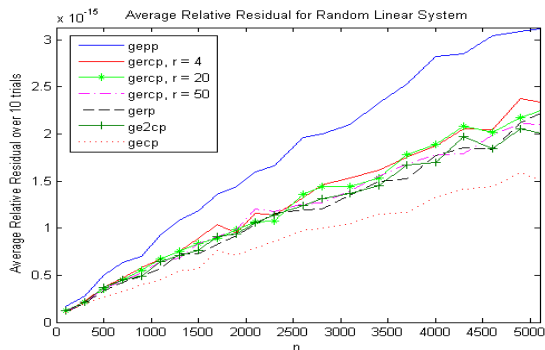
Numerical Experiments for RGECP (II)

► GEPP and Randomized GECP Element Growth



Numerical Experiments for RGECP (III)

- ▶ GEPP and Randomized GECP Residuals for random linear systems.



Numerical Experiments for Spectrum-revealing LU (I)

- ▶ SRLU vs. truncated LU, execution times.

Experiments: Benchmarking

Time comparison (seconds) with truncated DGETRF (truncated rank=300)
(rand orth)*(decaying diag)*(rand orth), numeric rank \approx 340

TR-LUCP (recursive column selection)					
		Cols			
		2000	4000	6000	8000
Rows	2000	0.090	0.163	0.236	0.309
	4000	0.153	0.253	0.357	0.460
	6000	0.215	0.344	0.477	0.605
	8000	0.271	0.429	0.589	0.754

DGETRF_TRUNC					
		Cols			
		2000	4000	6000	8000
Rows	2000	0.105	0.215	0.326	0.435
	4000	0.228	0.436	0.659	0.879
	6000	0.325	0.656	0.989	1.328
	8000	0.442	0.890	1.339	1.788

TIME RATIO					
		Cols			
		2000	4000	6000	8000
Rows	2000	86%	75%	73%	71%
	4000	67%	58%	54%	52%
	6000	66%	52%	48%	46%
	8000	61%	48%	44%	42%

Edison, single core, MKL

Numerical Experiments for Spectrum-revealing LU (II)

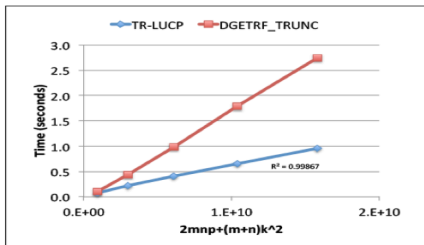
- ▶ SRLU vs. truncated LU, Complexity comparison.

Experiments: Scaling

Time comparison (seconds) with truncated DGETRF (truncated rank=300)
(rand orth)*(decaying diag)*(rand orth), numeric rank ≈ 340

TR-LUCP (recursive column selection)						
		Cols				
		2000	4000	6000	8000	10000
Rows	2000	0.076	0.145	0.219	0.289	
	4000	0.130	0.218	0.316	0.412	
	6000	0.170	0.290	0.413	0.557	
	8000	0.214	0.358	0.507	0.657	
	10000					0.958

DGETRF_TRUNC						
		Cols				
		2000	4000	6000	8000	10000
Rows	2000	0.105	0.215	0.326	0.435	
	4000	0.228	0.436	0.559	0.879	
	6000	0.325	0.656	0.989	1.528	
	8000	0.442	0.890	1.339	1.788	
	10000					2.738

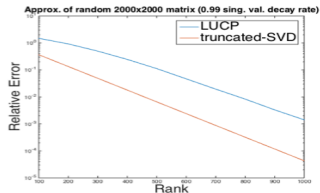
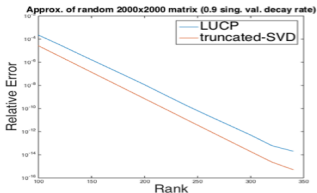
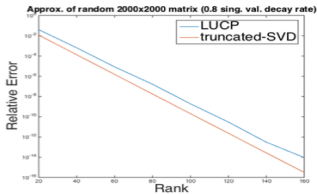
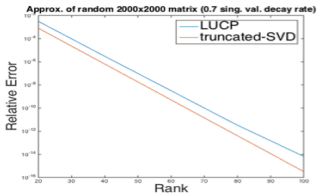


Edison, single core, MKL

Numerical Experiments for Spectrum-revealing LU (III)

- ▶ SRLU vs. truncated SVD.

Experiments: SVD-like Accuracy



Numerical Experiments for Spectrum-revealing QR (I)

- ▶ Edison (12 core, shared memory machine).
- ▶ Matrix A is 12000×12000 .
- ▶ `dgeqp2` (full decomposition), 75.84s
- ▶ `dgeqrf` (full decomposition), 5.91s
- ▶ truncated $k=1200$ randomized `qrqp`, 1.59s
- ▶ truncated $k=600$ randomized `qrqp`, 0.76s

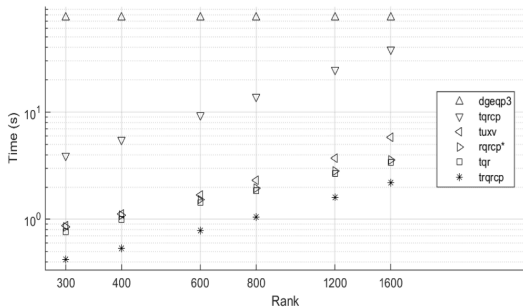
Numerical Experiments for Spectrum-revealing QR (II)

True BLAS-3 QRCP with Random Sampling and Low Rank Approx

Low-rank approximations

Performance of TRQRCP, rank scaling

Random 12000×12000 rank- r factorizations on 24-cores:



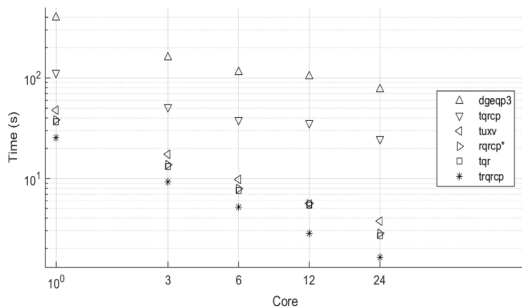
Numerical Experiments for Spectrum-revealing QR (III)

True BLAS-3 QRCP with Random Sampling and Low Rank Approx

Low-rank approximations

Performance of TRQRCP, parallel scaling

Random 12000×12000 rank-1200 factorizations on p -cores:

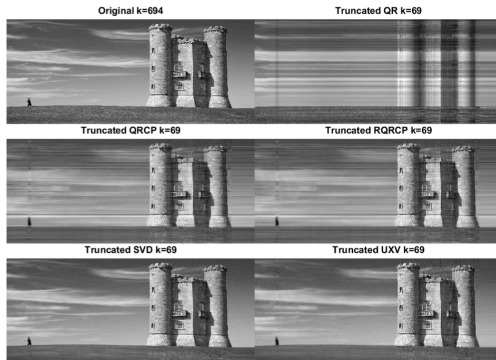


Numerical Experiments for Spectrum-revealing QR (IV)

True BLAS-3 QRCP with Random Sampling and Low Rank Approx

Approximate SVD

Tower



Conclusions

- ▶ A new suite of randomized algorithms for numerical linear algebra and low-rank matrix approximations, along with theoretical analysis.
- ▶ Codes in different stages of development.
- ▶ Communication-avoiding variants needed.

Thank you