From Becker-Döring to Lifshitz-Slyozov : deriving the boundary condition

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We investigate the connection between two classical models of phase transition phenomena, the (discrete size, infinite set of ODE) Becker-Döring equations and the (continuous size, PDE) Lifshitz-Slyozov equation [1]. For general coefficients and initial data, we introduce a scaling parameter and show that the empirical measure associated to the Becker-Döring system converges in some sense to the Lifshitz-Slyozov equation when the parameter goes to 0.

Contrary to previous studies, we use a weak topology that includes the boundary of the state space, allowing us to rigorously derive a boundary value for the Lifshitz-Slyozov model. It is the main novelty of this work and it answers to a question that has been conjectured or suggested by both mathematicians and physicists. We emphasize that the boundary value depends on a particular scaling (as opposed to a modeling choice) and is the result of a separation of time scale and an averaging of fast (fluctuating) variables.

We believe such procedure may be applied to various situation, when one tries to reduce the complexity of models, as in individual-based modeling for instance. Such results have several applications on its own, too. First, it allows to have fast numerical approximations of the Becker-Döring model. Then, as we used a stochastic version of the Becker-Döring model, large deviations around the limiting model may give a satisfactory notion of the "nucleation time", defined as the time needed to obtain a large aggregate (work in progress).

1. Preprint available at http ://arxiv.org/abs/1412.5025