Modeling and estimation for Gaussian fields indexed by graphs, application to road traffic prediction

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Framework

Road traffic (Médiamobile) :

- Activity: Real-time prediction of traveling time
- Aim: Understand the speed process on the road traffic network
- Observations :
  - Fixed sensors: corrupted values
  - Cars fleet: unobserved areas
  - The graph is known
- Probem: Use the spatial dependency for:
  - Spatial completion
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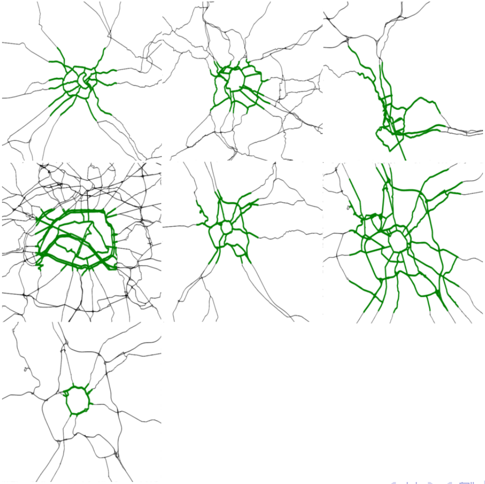
Modeling : Random process $(X_i^{(n)})_{n\in\mathbb{Z},i\in G}$



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Objective: Yield a parametric model $(\mathcal{K}_\theta)_{\theta\in\Theta}$ for covariance operators of X

Outline

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Graph

Model: Zero-mean Gaussian field $(X_i)_{i \in G}$ indexed by the vertices G of a graph **G**.

Definition (Unoriented weigthed graph)

$\mathbf{G} = (G, W)$:

- *G set of vertices (countable)*
- $W \in [-1, 1]^{G \times G}$ *Weighted adjacency operator (symmetric)*

Neighbors: $i \sim j$ if $W_{ij} \neq 0$
Degree of a vertex: $D_i = \sharp \{j, i \sim j\}$.

Assumption (H_0)

- $D := \sup_{i \in G} D_i < +\infty$, **G** *has bouded degree*
- $\forall i \in G, \sum_{j \in G} |W_{ij}| \leq 1$ *even renormalizing*

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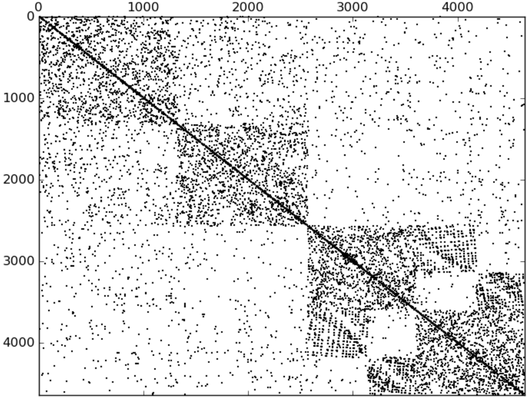
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Problem

Speed of vehicles on the road network **at a fixed time**:
zero-mean Gaussian field $(X_i)_{i \in G}$ indexed by the vertices of a graph.

Aim: Chose a model for covariance operators

Modeling constraints

- Adaptability to physical modeling
- Compatibility with classical cases (time series, $\mathbb{Z}^d$, homogeneous tree...)
- Extension of classical tools from time series (spectral representation, Whittle's estimation...)

$\Rightarrow$ Define covariance operators from a spectral construction

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Modeling for covariance operators

Models for covariance operators (of the speed field)

$$\mathcal{K}(f) = f(W)$$

Example : Finite dimension

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W acts on $l^2(G)$:

$$\forall u \in l^2(G), \forall i \in G, (Wu)_i := \sum_{j \in G} W_{ij} u_j.$$

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Under H_0

W is a bounded Hilbertian self-adjoint operator in $B_G := l^2(G) \to l^2(G)$:

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Spectral decomposition

$$\exists E, \mathcal{M}, W = \int_{\mathcal{M}} \lambda \mathrm{d}E(\lambda)$$

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Definition (Identity resolution)

$\mathcal{M}$ σ-algebra $E : \mathcal{M} \to B_G$ such that $\forall \omega, \omega' \in \mathcal{M}$,

1. $E(\omega)$self-adjoints projectors.
2. $E(\emptyset) = 0, E(\Omega) = I$
3. $E(\omega \cap \omega') = E(\omega)E(\omega')$
4. Si $\omega \cap \omega' = \emptyset$, alors $E(\omega \cup \omega') = E(\omega) + E(\omega')$

Models for covariance operators, spectral density

Definition (Construction of the covariance operators)

Let g be an positive function, analytic over $\mathrm{Sp}(W)$,

$$\mathcal{K}(g) = \int_{\mathrm{Sp}(W)} g(\lambda)\mathrm{d}E(\lambda),$$

- *g polynomial:* $MA_q^{(W)}$
- $\frac{1}{g}$ *polynomial:* $AR_p^{(W)}$ ...

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Remarks:

- $\mathcal{K}(g) = g(W)$ (Normal convergence of the *PSD*)
- Dependency in W
- Analogy with $\mathbb{Z}$

Local measures, trace measure

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$$\forall i, j \in G, \forall \omega \in \mathcal{M}, \mu_{ij}(\omega) = E_{ij}(\omega).$$

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Definition (Trace measure associated to G_n)

$$\frac{1}{\sharp G_n} \sum_{g \in G_n} \mu_{gg}$$

$G = \mathbb{Z}$: compatibility with time series

Adjacency operator

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$$\forall i, j \in G, \forall k \in \mathbb{Z}, \left(W^k\right)_{ij} = \frac{1}{\pi}\int_{[-1,1]} \lambda^k \frac{T_{|j-i|}(\lambda)}{\sqrt{1-\lambda^2}} \mathrm{d}\lambda.$$

T_k : $k^{\text{ième}}$ Chebychev polynomials

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Spectral density

$$f(t) = g(\cos(t))$$

$$\mathcal{K}(g)_{ij} = \frac{1}{2\pi}\int_{[-\pi,\pi]} f(t)\cos\left((j-i)t\right)\mathrm{d}t := \left(\mathcal{T}(f)\right)_{ij}$$

An example

Dynamic model of spatio-temporal speed agregation:

$$\forall n \in \mathbb{Z}, i \in G, X_i^{(n+1)} = \epsilon_i^{(n+1)} + aX_i^{(n)} + b \sum_{j \sim i} X_j^{(n)}.$$

Covariance operator

Under stationarity:

$$\mathrm{Cov}(X) = \sum_{n \geq 0} \sum_{k \leq n} \binom{n}{k} W_{\mathbb{Z}}^k \otimes \left(a \, \mathrm{Id}_G + bW_G\right)^{2n-k}$$

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Outline

The concrete problem

Problem:

- $\Theta \subset \mathbb{R}$ compact
- $(f_\theta)_{\theta \in \Theta}$ parametric family of spectral densities associated to $\mathcal{K}(f_\theta) = f_\theta(W)$
- Asymptotic on $(\mathbf{G}_n)_{n \in \mathbb{N}}$ sequence of nested subgraphs inducted by $\mathbf{G}$
  Example $G = \mathbb{Z}$: $G_n = [1, n]$.
- $\theta_0 \in \mathring{\Theta}$, $\mathbf{X} \sim \mathcal{N}\left(0, \mathcal{K}(f_{\theta_0})\right)$
- We observe the restriction X_n of $\mathbf{X}$ to $\mathbf{G}_n$, cov : $\mathcal{K}_n(f_\theta)$
- $m_n = \sharp G_n$

Aim: Estimate θ_0 with a maximum likelihood method:

$$L_n(\theta) := -\frac{1}{2}\left(m_n \log(2\pi) + \log\det\left(\mathcal{K}_n(f_\theta)\right) + X_n^T \left(\mathcal{K}_n(f_\theta)\right)^{-1} X_n\right)$$

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Computational issues: Maximize an approximation of the log-likelihood

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$$\frac{1}{n}\log\det\left(\mathcal{T}_n(f_\theta)\right) \to \frac{1}{2\pi}\int_{[0,2\pi]}\log\left(f_\theta\right)\mathrm{d}t.$$

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De plus, $\mathcal{T}(f)\mathcal{T}(g) = \mathcal{T}(fg)$, d'où $(\mathcal{T}(f))^{-1} = \mathcal{T}(\frac{1}{f})$

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Whittle's approximation for $\mathbb{Z}$, log det

$$\frac{1}{n} \log\det\left(\mathcal{T}_n(f_\theta)\right) \to \frac{1}{2\pi}\int_{[0,2\pi]} \log\left(f_\theta\right) \mathrm{d}t.$$

Whittle's approximation for $\mathbb{Z}$, periodogram

$$\frac{1}{n}\left(X_n^T \left(\mathcal{T}_n(f_\theta)\right)^{-1} X_n - X_n^T \mathcal{T}_n\left(\frac{1}{f_\theta}\right) X_n \right) \to 0, \text{ p.s.}$$

Classical case $\mathbb{Z}$

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$$\tilde{L}_n(\theta) := -\frac{1}{2}\left(m_n \log(2\pi) + \frac{1}{2\pi}\int_{[0,2\pi]} \log\left(f_\theta\right) \mathrm{d}t + \frac{1}{n} X_n^T \mathcal{T}_n(\frac{1}{f_\theta}) X_n\right)$$

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log det approximation for a graph

Assumption (Existence of the limit trace measure)

H_1 : $\exists \mu, \frac{1}{m_n} \sum_{g \in G_n} \mu_{gg} \to \mu$

Assumption (Edge effects)

H_2 : $\delta_n = o(m_n)$

Whittle's approximation for G, log det

$$\frac{1}{m_n} \log \det (\mathcal{K}_n(f_\theta)) \to \int_{\mathrm{Sp}(W)} \log (f_\theta) \, \mathrm{d}\mu(t)$$

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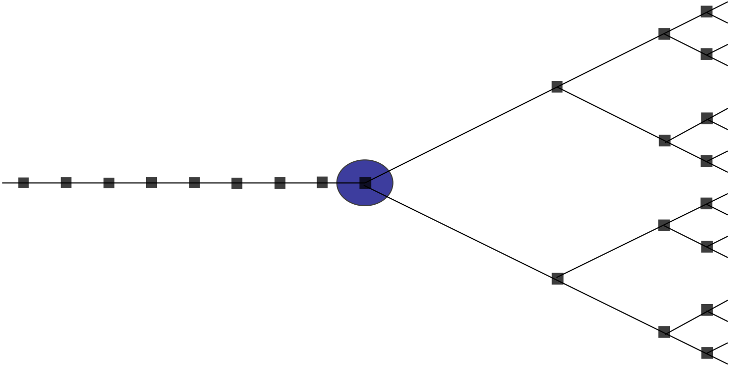
Sufficient conditions for the existence of the trace measure μ



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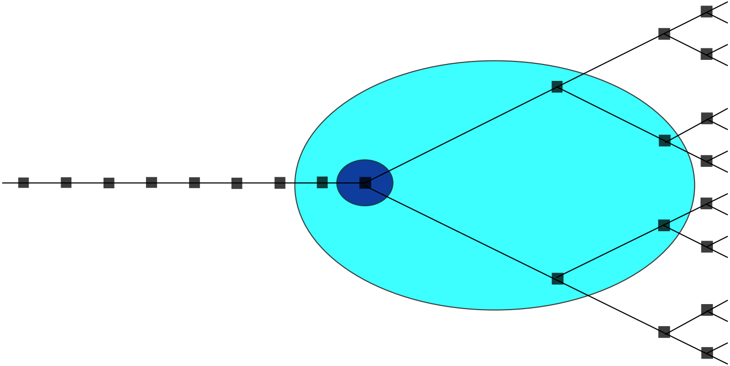

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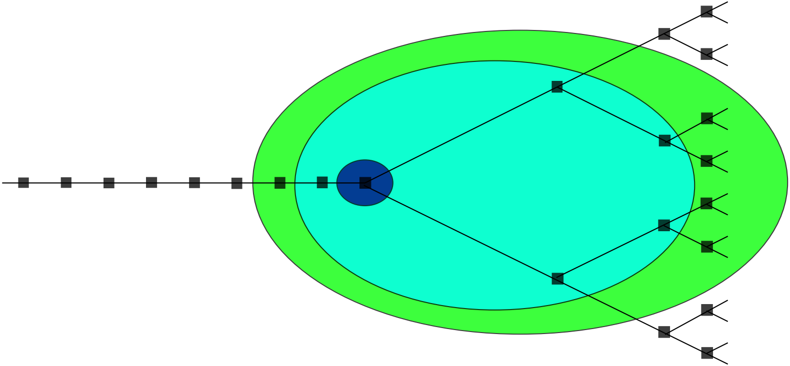

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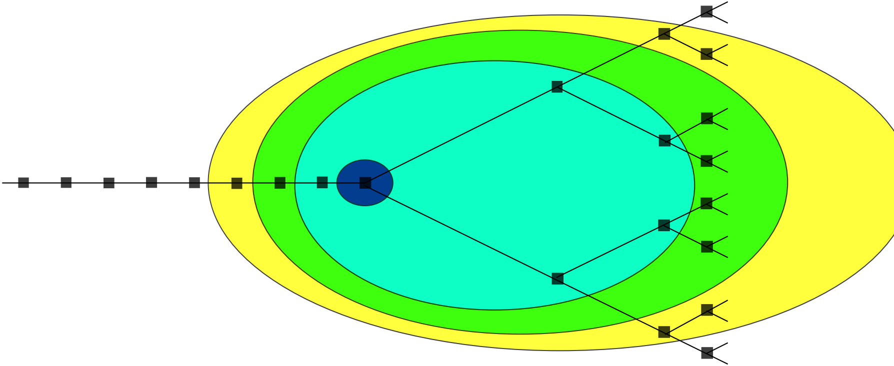

Consistency

Let $\theta_n, \bar{\theta}_n, \tilde{\theta}_n$ resp. arg max of

$$
\begin{aligned}
L_n(\theta) &:= -\frac{1}{2}\left(m_n \log(2\pi) + \log\det\left(\mathcal{K}_n(f_\theta)\right) + X_n^T \left(\mathcal{K}_n(f_\theta)\right)^{-1} X_n\right) \\
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Assumption (H_3)

- $\theta \to f_\theta$ *injective*
- $\forall \lambda \in Sp(W), \theta \to f_\theta(\lambda)$ *continuous.*
- *Strong regularity assumptions on* $(f_\theta)_{\theta\in\Theta}$

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Theorem (Consistancy of the Whittle's estimate)

The estimators θ_n,$\bar{\theta}_n$, $\tilde{\theta}_n$ converge $P_{f_{\theta_0}}$-a.s. to the true value θ_0.

Asymptotic normality and efficiency

Tappered likelihood

$$-2L_n^{(u)}(\theta) := m_n \log(2\pi) + m_n \int \log(f_\theta(x))\mathrm{d}\mu(x) + X_n^T \left(\mathcal{Q}_n(\frac{1}{f_\theta}) \right) X_n.$$

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$$\theta_n^{(u)} = \arg\max L_n^{(u)}.$$

Asymptotic normality and efficiency

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Theorem (Asymptotic normality)

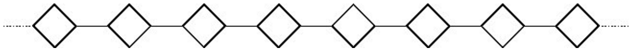
For $\theta_0 \in \mathring{\Theta}$, in the AR_L or MA_L cases, and under assumptions on the graph and the family of spectral densities,$\theta_n^{(u)}$ converges to θ_0, and is asymptotically normal and efficient:

$$\sqrt{m_n}(\theta_n^{(u)} - \theta_0) \to \mathcal{N}\left(0, \left(\frac{1}{2} \int \frac{(f'_{\theta_0})^2}{f_{\theta_0}^2} \mathrm{d}\mu \right)^{-1} \right).$$

Outline

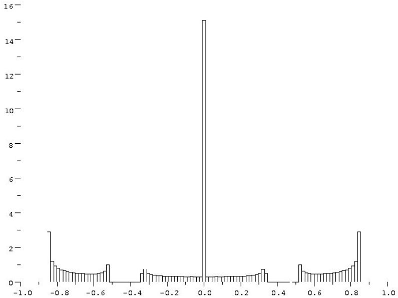
Applications

Figure: Graphe G



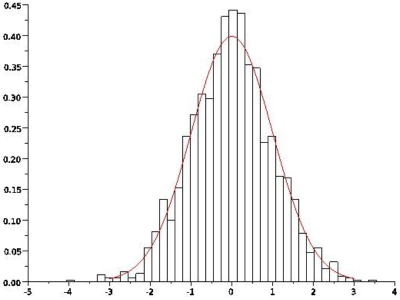
Applications

Figure: Empirical spectral measure

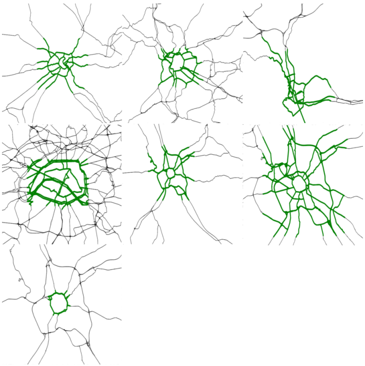


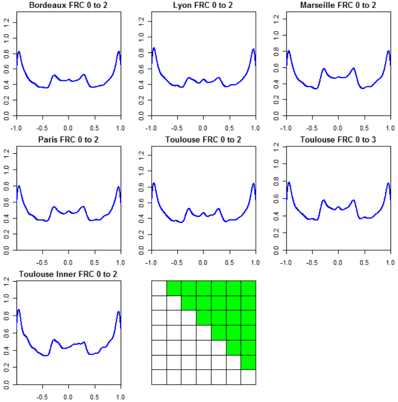
Applications

Figure: Empirical distribution of estimation error



Spectrum of the road network





Projects

In progress

- Choice/estimation of the generator
- Spectral study and modeling of the road network
- Random process indexed by random graphs

Future works ?

- Link with physicals models
- Use approximation of manifolds by graphs
- Extension of the notion of causality

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Merci !

Normalité asymptotique et efficacité

Cadre :

- AR_L
- MA_L

L-type pour les couples de sommets

$$g, h \in G, t^{(L)}(g,h) = (W^l_{gh})_{l=0,\cdots,L}$$

$$\begin{aligned} B^{(n,L)}_{ij} &:= \frac{\text{Card}\left\{(k,l) \in G_n \times G, \mu_{kl} = \mu_{ij}\right\}}{\text{Card}\left\{(k,l) \in G_n \times G_n, \mu_{kl} = \mu_{ij}\right\}}, d(i,j) \leq L \\ &:= 1 \text{ si } d(i,j) > L. \end{aligned}$$

Exemple $G = \mathbb{Z}$: $B^{(n,L)}_{i,i+k} = \mathbf{1}_{k \leq L} \frac{n}{n-k}$

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Forme "tordue"

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Exemple $G = \mathbb{Z}$: $X^T \mathcal{Q}_n(1) X$ covariance empirique non-biaisée

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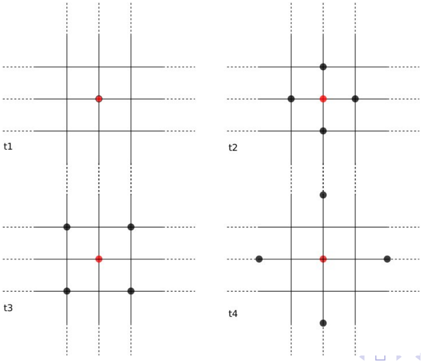

Limites du cadre précédent

Double constat

- Les processus construits analytiquement sont réguliers
- Sur un graphe distance-transitif, $MA^{(W)}$, $MA^{(L)}$, $MA^{(\tilde{L})}$ ont même sens

Objectifs

- Choisir les modifications de graphes A admissibles pour les $MA^{(A)}$ (quitte à renormaliser)
- Etendre à des procesus non réguliers (sans représentation MA)

$\Rightarrow$ Définition de stationnarité "algébrique" plutôt qu'analytique

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Idée : Proposer un modèle de covariance “stationnaire” et “isotrope” généralisant les notions suivantes :

- Sur $\mathbb{Z}$: invariance par translation
- Sur $\mathbb{Z}^d$: invariance par rotations/translations
- Arbres homogènes, graphes de Cayley... : invariance par automorphisme de graphe

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Démarche

Propriétés souhaitées

- Covariance dans $F_G : l^1(G) \to l^\infty(G)$
- Invariance par automorphisme si il en existe
- Contient fonctions positives de W, D opérateur de degrés, $L = D - W$ Laplacien discret...
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Idée

Construire opérateurs de covariance comme image de W par une classe bien choisie de fonctions “invariantes”

Stationnarité

Definition (Fonctions invariantes I_G)

Φ *de* $\mathrm{Dom}(\Phi) \subset B_G$ *dans* F_G *invariante si*

- $\forall \sigma \in \mathcal{S}, \forall W \in \mathrm{Dom}(\Phi), W \circ \sigma \in \mathrm{Dom}(\Phi), W^T \in \mathrm{Dom}(\Phi)$
- $\forall \sigma \in \mathcal{S}, \forall W \in \mathrm{Dom}(\Phi), \Phi(W \circ \sigma) = \Phi(W) \circ \sigma$
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Remarque : Conditions de symmétrie sur les variables

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Remarque :

- Compatibilité avec définitions existantes
- Contient L,$\tilde{L}$, $W, \cdots$

Soit $A = \Phi(W), \Phi \in I_G$, tel que

$$W_{ij} = 0 \Rightarrow A_{ij} = 0, \left|A_{ij}\right| \leq 1$$

$\Rightarrow A$ modification "isotrope" du graphe $\Rightarrow MA^{(A)}$

Exemples :

- Laplacien
- Laplacien renormalisé
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