Book Review

J. Frédéric Bonnans, J. Charles Gilbert, Claude Lemaréchal, and Claudia A. Sagastizábal: Numerical Optimization, Springer (2003) ISBN 3-540-00191-3
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This book is concerned with algorithms for continuous optimization problems. It is split into four large parts: Preliminaries (19 pages), Unconstrained Optimization (75 pages), written by Claude Lemaréchal, Nonsmooth Optimization (58 pages), by Claudia A. Sagastizábal, Newton's Method in Constrained Optimization (134 pages), by J. Charles Gilbert, and Interior Point Algorithms for Linear and Quadratic Optimization (106 pages), by J. Frédéric Bonnans. Each part starts with some general remarks, explaining the problem at hand, usually followed by a chapter with the theoretical results needed. Many chapters finish with historical remarks, some even with exercises. The book closes with an index and a list of 319 references, thereby covering the most important part of the research literature.

The preliminaries give some general introduction into the field of optimization, mainly by describing several real-world application problems that the authors have encountered. Then, fundamental solution principles and problems are discussed, all in very accessible style.

The rest of the book is, as it has been outlined above, divided into four parts, each of which written by a different author. These parts in itself cover a large part of what is considered continuous optimization nowadays, and present the reader also with an inkling of what is going on in contemporary research. Not counting some remarks dispersed in the text here and there, these parts are not interconnected with each other and are therefore self-contained. This is certainly an advantage if one is interested in only one of the topics at a time. Only if the book is run through from cover to cover, one sees that a closer mingling of the parts might be worthwhile. For example, large parts of the section on duality for linear programs (Chapter 17) might also occur, in a slightly more general form, in Chapter 7 (Some Theory of Nonsmooth Optimization), where they fit nicely in the subsection on duality. Indeed, the interconnection between the different parts is so weak that sometimes the notation changes, albeit slightly, from one part to the next. For example, sometimes the standard dot product between two vectors x and y is denoted by (x,y), sometimes by $x \cdot y$, but usually by $x^\top y$. Likewise, the Euclidean norm is usually denoted by $\|\cdot\|$, but sometimes by $|\cdot|$. These are small qualms, however, and they should not be too distracting to a prospective reader. Nevertheless, the book consists, at least in principle, of four different smaller books, and it is therefore fitting to discuss each of them separately.

Part II, Unconstrained Optimization, presents a review of Newton's method, BFGS, CG, Trust Region Methods, and the necessary details of line searches. All the most important issues with respect to unconstrained optimization are therefore treated. However, Trust Region methods are presented rather tersely.

Part III, Nonsmooth Optimization, discusses, among others, dual problems as one source of nonsmoothness, as well as decomposition techniques. The chapter on bundle methods is particularly beautiful, deriving the main result without too many technicalities.

Part IV, Newton's Method in Constrained Optimization, gives a thorough treatment of one of the most important techniques for smooth constrained optimization, the SQP algorithm. It is the largest part of the book, and, given the technical difficulties and the host of results to cover, rightly so.

Part V, Interior Point Algorithms for Linear and Quadratic Optimization, discusses in detail the theoretical underpinnings of interior-point algorithms for the cases mentioned. First, the simplex method and some duality theory is rederived. The text then moves rapidly on to linear monotone complementarity problems, in order to cast both problem types discussed in this framework. Then, a predictor-corrector as well as a large neighbourhood algorithm are derived, the latter appearing to be new. Non-feasible and one-step algorithms are discussed, and the problem of starting points is tackled.

All in all, this is a worthwhile book to buy and to read, if one is interested in continuous optimization techniques. The goal of the book is an elucidation of the theoretical workings and underpinnings of the main algorithmic tools in use nowadays, and it succeeds in that. However, details on the numerical analysis necessary to turn these results into working computer programs and references to actual computer codes are sadly lacking. This is especially true for Part IV and V, where a discussion of the numerical linear algebra, one of the main building blocks and one of the main sources of difficulties for succesful practical algorithms, is done in a few terse remarks. One has to turn to other resources for a detailed treatment of these issues.

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