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HUMAN ELEMENT

SESSION ID: **CRYP-T08**

Generic Attack on Iterated Tweakable FX Constructions

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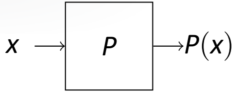
Introduction

Permutation

A bijective pseudorandom function.

$P : \{0,1\}^n \rightarrow \{0,1\}^n$

Example: Keccak-f

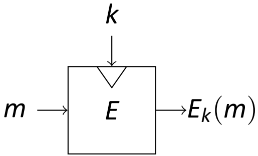


Block Cipher

A family of permutations indexed by a (secret) key.

$E : \{0,1\}^\kappa \times \{0,1\}^n \rightarrow \{0,1\}^n$

Example: AES, DES



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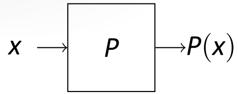

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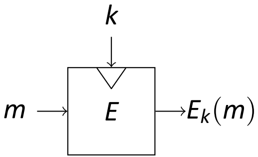
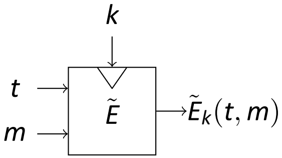


Tweakable Block Cipher

A family of permutations indexed by a key and a (public) tweak.

$\tilde{E} : \{0,1\}^\kappa \times \{0,1\}^\tau \times \{0,1\}^n \to \{0,1\}^n$

Example: Deoxys, Skinny



Introduction

All those primitives are used for Authenticated Encryption.

- Permutation: Sponge based modes (Monkey duplex, Beetle, ...)
- Block Cipher: Most common (GCM, CCM, ...)
- Tweakable Block Cipher: Needed for analysis of OCB, XTS, PMAC, ...

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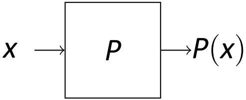
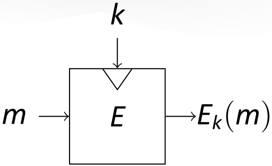
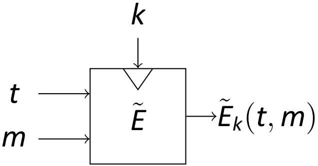
2-Step Proofs

First prove a mode is secure using a Tweakable Block Cipher.
Then build a Tweakable Block Cipher from an existing Block Cipher.

Introduction







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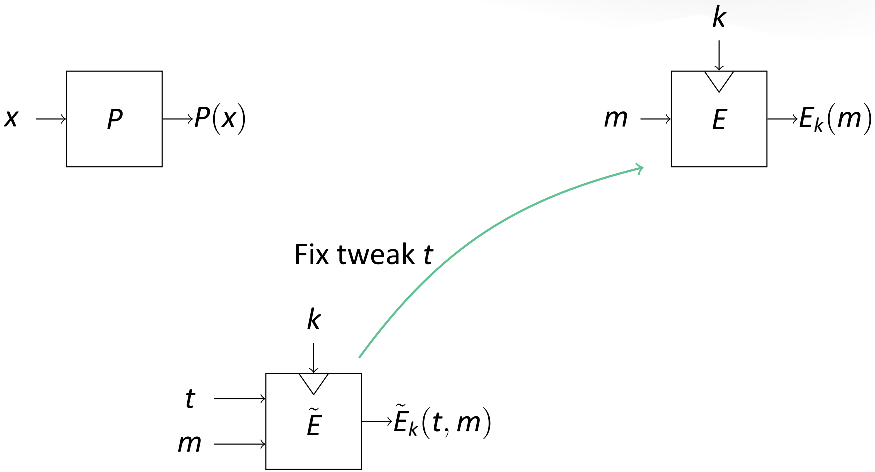

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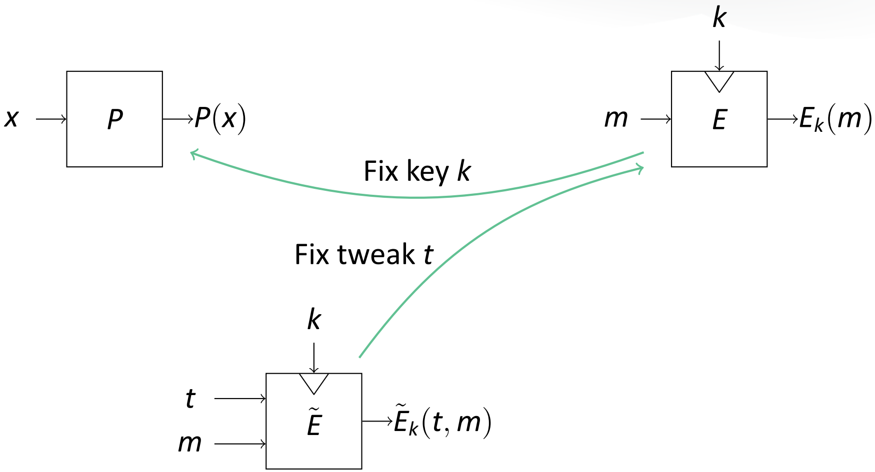

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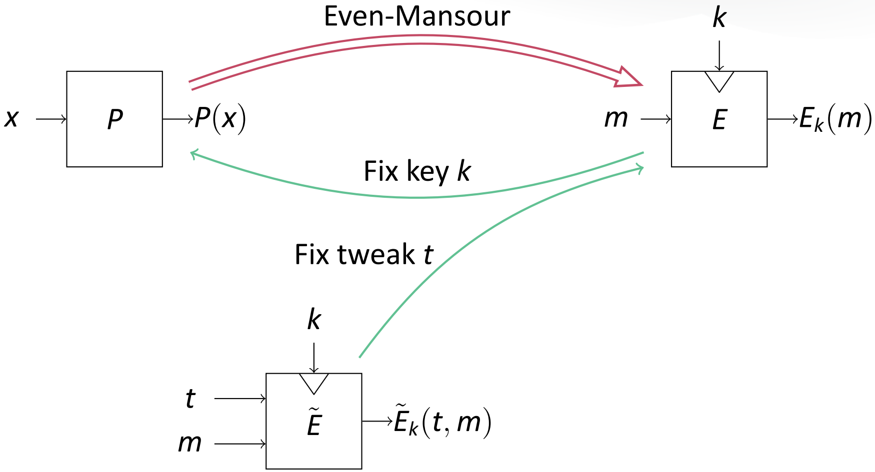

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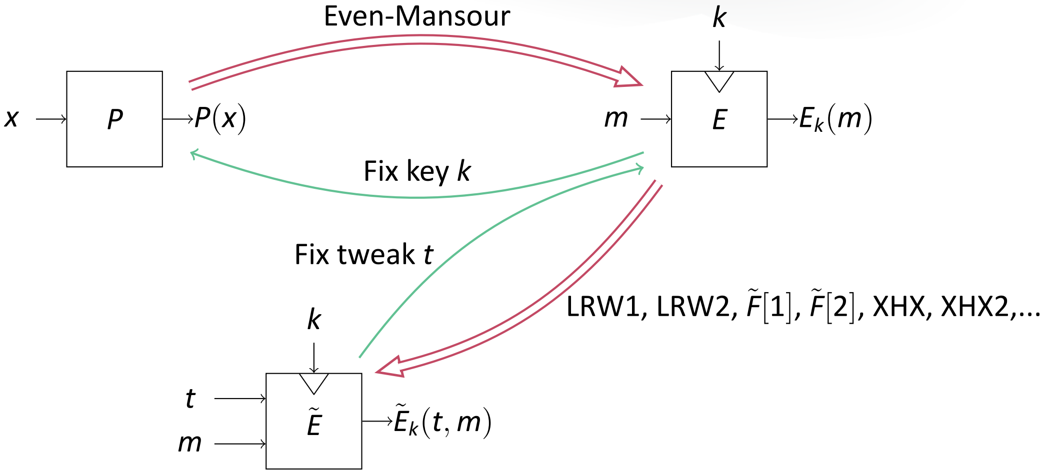
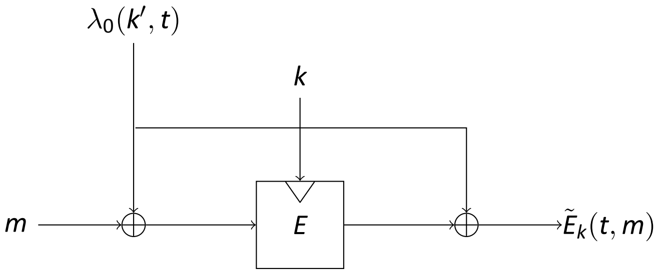


LRW2 [Liskov, Rivest, Wagner, 2011]

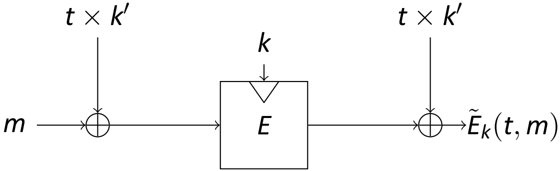
It uses:

- 1 n-bit AXU function $\lambda_0(k', t)$.
- 2 secret values k, k'.



Secure Tweakable Block Cipher up to $2^{n/2}$ calls.

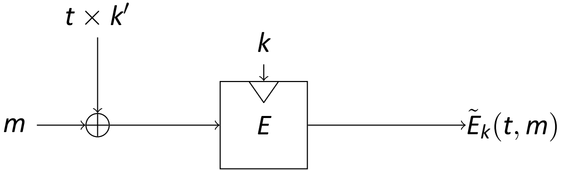
XEX[Rogaway, 2004]



Uses Galois field multiplication $t \times k'$ for a secret value k'.
Preserves CCA security.

Secure Tweakable Block Cipher up to $2^{n/2}$ calls.

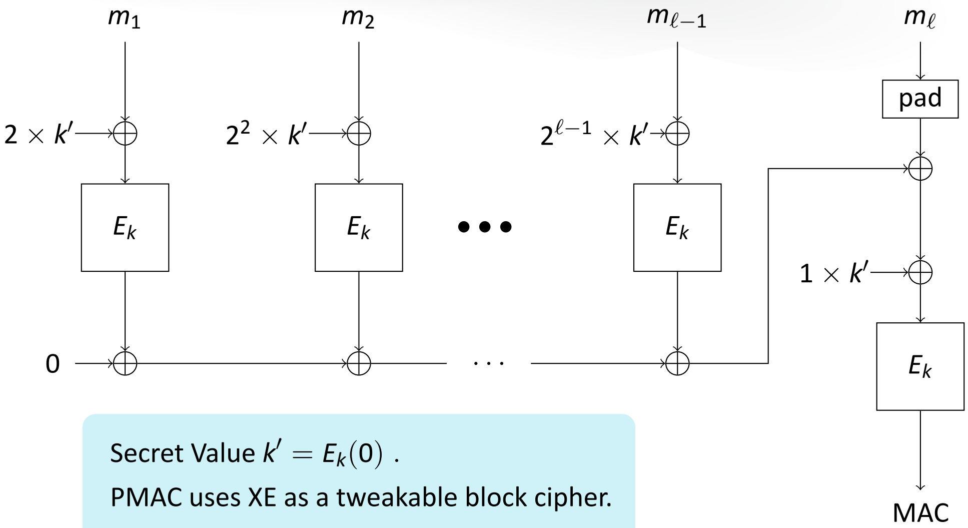
XE [Rogaway, 2004]



Uses Galois field multiplication $t \times k'$ for a secret value k'.
Preserves CPA security.

Secure Tweakable Block Cipher up to $2^{n/2}$ calls.

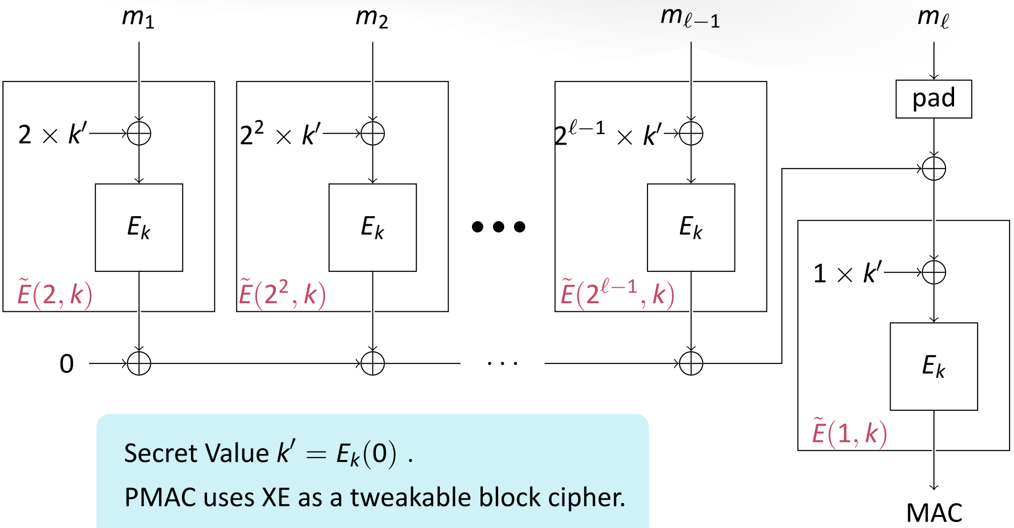
2-step proof for PMAC



Secret Value $k' = E_k(0)$.

PMAC uses XE as a tweakable block cipher.

2-step proof for PMAC



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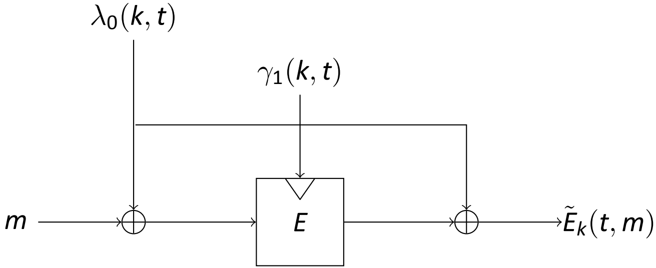
PMAC uses XE as a tweakable block cipher.

XHX[Jha, List, Minematsu, Mishra, Nandi]

It uses:

- 1 n-bit subkey $\lambda_0(k,t)$.
- 1 κ-bit subkey $\gamma_1(k,t)$.

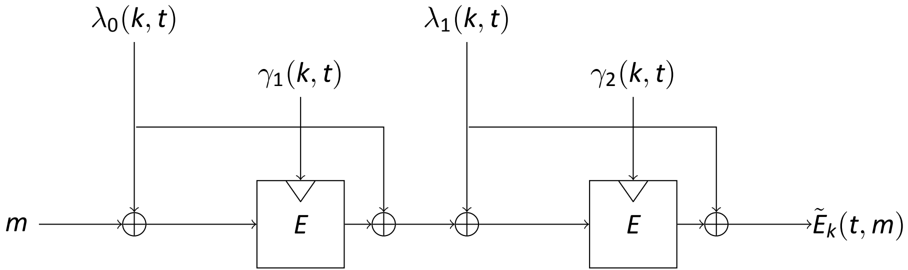


Typically λ_0 and γ_1 can use field multiplication with a secret derived with k.
Allowing rekeying improves the security up to $2^{\frac{n+\kappa}{2}}$.

XHX2 [Lee, Lee]

It uses:

- 2 n-bit subkeys $\lambda_0(k,t)$, $\lambda_1(k,t)$.
- 2 κ-bit subkeys $\gamma_1(k,t)$, $\gamma_2(k,t)$.

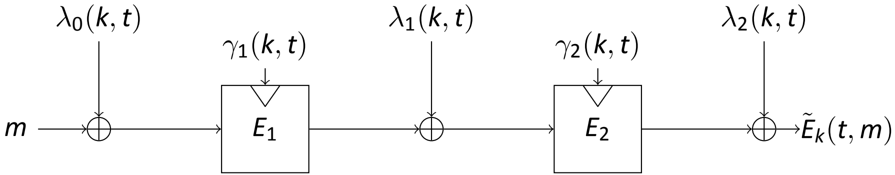


Cascade of two independant XHX.
Cascading improves the security up to $2^{\frac{2}{3}(n+\kappa)}$.

2-Round Tweakable FX

It uses:

- 3 n-bit subkeys $\lambda_0(k,t)$, $\lambda_1(k,t)$, $\lambda_2(k,t)$.
- 2 κ-bit subkeys $\gamma_1(k,t)$, $\gamma_2(k,t)$.



Generalization

We don't assume anything on subkey functions.

$\Longrightarrow$ Attack works for any 2-round schemes !

Information Theoretic Setting

Proofs say an attacker needs at least this much data.
Proofs can get better, it is a lower bound.
Information Theoretic cryptanalysis shows an upper bound on the provable security.
A proof is tight when cryptanalysis matches.
Computations are irrelevant.

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Information Theoretic Key Recovery

It's all about the query complexity.

We count calls to tweakable block cipher $\tilde{E}_k(\cdot,\cdot)$ and block ciphers $E_1(\cdot,\cdot), E_2(\cdot,\cdot)$.

Computation of subkey functions are not counted.

GOAL: Recover the master key k.

Our Result

How far can we hope to go by cascading and rekeying?

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This work

Information theoretic cryptanalysis.

Query complexity of $\mathcal{O}(2^{\frac{r}{r+1}(n+\kappa)})$.

Show that XHX and XHX2 proofs are tight.

Our Strategy

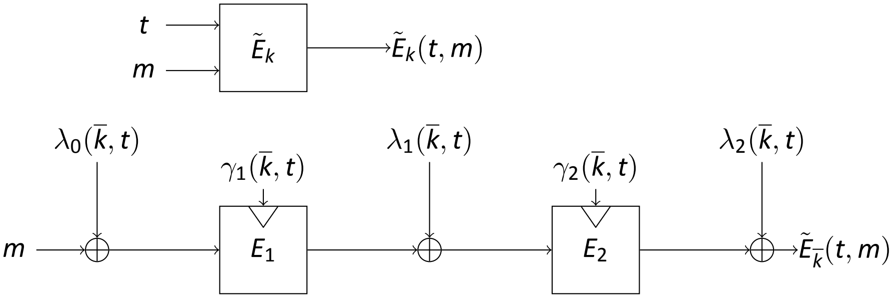
We follow the same strategy as [Gaži, 2013] to improve and apply it in the tweakable block cipher setting.

Strategy

Build a contradictory path for each wrong key guesses until one is left.

Contradictory Path

1. Query $c = \tilde{E}_k(t, m)$ for some (t, m).
2. Make a guess $\overline{k}$ of the master key k.
3. Compute $\overline{c} = \tilde{E}_{\overline{k}}(t, m)$.
4. If $c \neq \overline{c}$ then Contradictory Path then $\overline{k} \neq k$.



Counting queries

- No issue with guessing all the keys in information theoretic setting.
- However we can't make a block cipher query for each guess, it's too much !
- We need to store and reuse previous queries as much as we can.

Tweakable Block Cipher

As we can have security $\gg 2^n$ we also can have online queries $\gg 2^n$!

Notations

- n and κ the block ciphers state and key size respectively.
- ℓ_0 the number of online queries to $\tilde{E}_k(t, m)$.
- ℓ the number of offline queries to $E(\overline{k}, m)$..

Total Asymptotic Query Complexity is $\mathcal{O}(\ell_0 + \ell)$.

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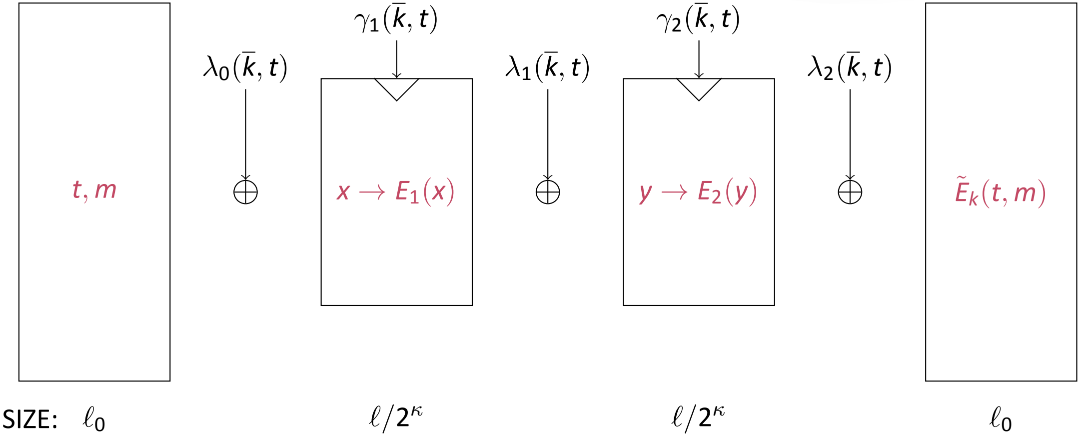
Total Asymptotic Query Complexity is $\mathcal{O}(\ell_0 + \ell)$.

Non-Adaptative Known Plaintext Attack

Observed ℓ_0 tweak/plaintext/ciphertext triples.
Compute random $\ell/2^\kappa$ input/output of block ciphers under each κ-bit subkey.

Random Path Reconstrution for 2 Rounds



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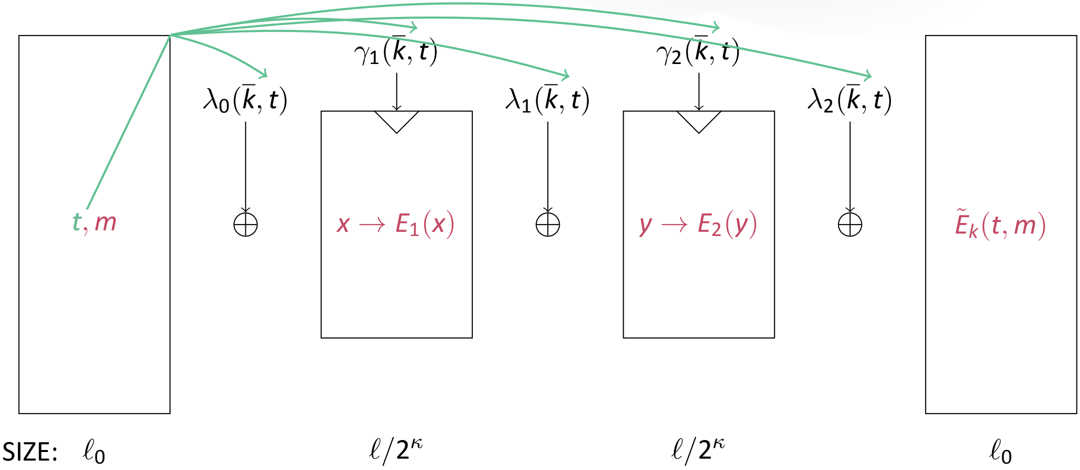

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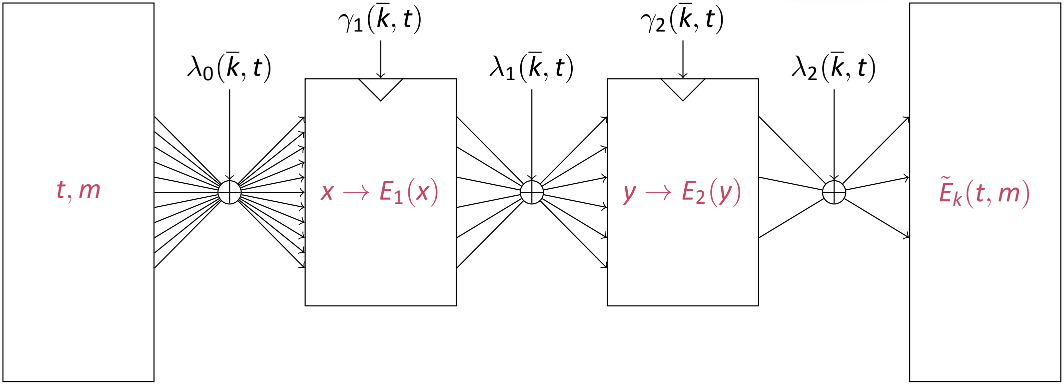

SIZE: ℓ_0 $\ell/2^\kappa$ $\ell/2^\kappa$ ℓ_0

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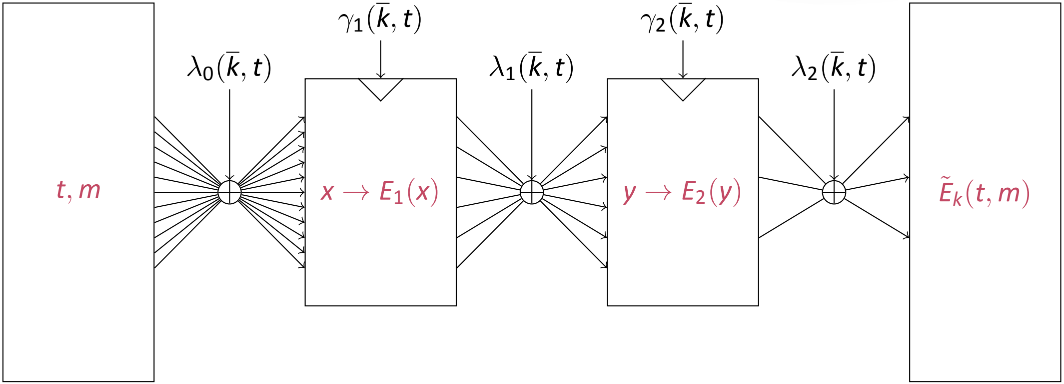

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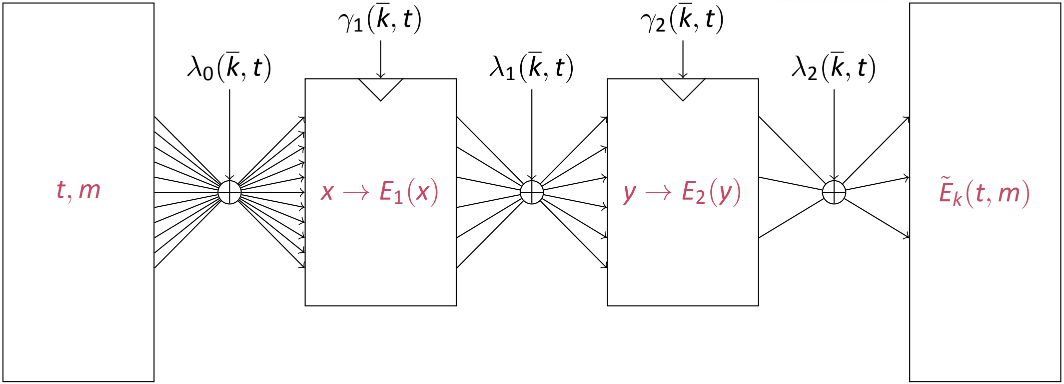

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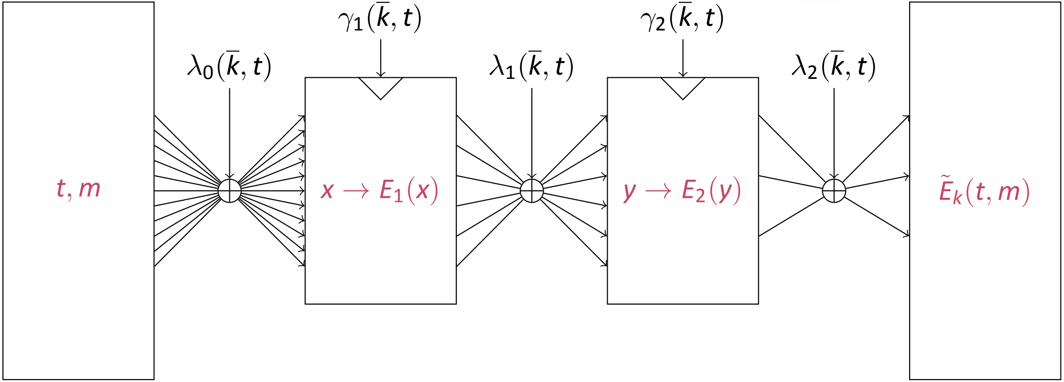

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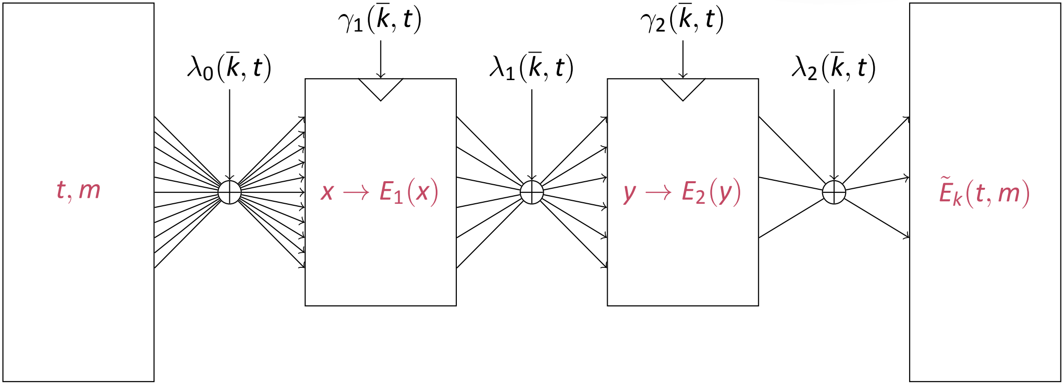

SIZE:	ℓ_0		$\ell/2^\kappa$		$\ell/2^\kappa$		ℓ_0
#PATH:		ℓ_0		$\ell_0\ell/2^{\kappa+n}$		$\ell_0\ell^2/2^{2\kappa+2n}$	$\bar{k} = k$?

Query Complexity

The number of path we can reconstruct is $\ell_0\ell^2/2^{2\kappa+2n}$ on average for all guesses $\overline{k}$.
We put $\ell_0 = \ell$ to minimize $\ell_0 + \ell$.

$$
\begin{aligned}
\ell_0\ell^2/2^{2\kappa+2n} &= 1\\
\ell^3/2^{2\kappa+2n} &= 1\\
\ell^3 &= 2^{2\kappa+2n}\\
\ell &= 2^{\frac{2}{3}(\kappa+n)} = \ell_0
\end{aligned}
$$

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Result

The query complexity of the attack is $\mathcal{O}(2^{\frac{2}{3}(\kappa+n)})$.

Parameter Constraint

There is no issue with having $\ell_0 > 2^n$ as the tweak can be of arbitrary size. However we need $\ell/2^\kappa \geq 1$ for our previous reasoning to hold.

$$\begin{aligned} \ell/2^\kappa &\geq 1 \\ 2^{\frac{2}{3}(\kappa+n)}/2^\kappa &\geq 1 \\ \frac{2}{3}\kappa + \frac{2}{3}n - \kappa &\geq 0 \\ -\kappa + 2n &\geq 0 \\ \kappa &\leq 2n \end{aligned}$$

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Constraint

Cryptanalysis works when the block cipher key size is less or equal to twice the state size.

Generalization for *r* rounds

The attack works for any number r of rounds.

Result

The query complexity of the attack is $\mathcal{O}(2^{\frac{r}{r+1}(\kappa+n)})$.

Constraint

Cryptanalysis works when $\kappa \leq rn$.

Technical Details

Need to ensure that the right key *k* is detected while all the wrong guesses be dismissed.
Possible false positive when the master key *k* is large !

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Need to ensure that the right key k is detected while all the wrong guesses be dismissed.
Possible false positive when the master key k is large !
Let k be a $\tilde{\kappa}$-bit value then:

Affined query complexity

The asymptotic query complexity is $\mathcal{O}(2^{\frac{r}{r+1}(n+\kappa)} \cdot \sqrt[r+1]{\tilde{\kappa}/n})$.

It is still $\mathcal{O}(2^{\frac{r}{r+1}(n+\kappa)})$ whenever $\tilde{\kappa}$ is a multiple of n.

Each tweak must give different subkey values for this key recovery to work but if not, then, we have a distinguisher.

Results

Ref	Scheme	r	Proof	Known Attack	Our Generic Attack
[LisRivWag11]	LRW2	1	$2^{n/2}$	$2^{n/2}$	$2^{\frac{1}{2}(n+\kappa)}$
[Mennink15]	$\tilde{F}[1]$	1	$2^{\frac{2}{3}n}$	2^n	2^n (as $\kappa = n$)
[Mennink16]	XPX	1	$2^{n/2}$	$2^{n/2}$	$2^{n/2}$ (as $\kappa = 0$)
[JLMMN17]	XHX	1	$2^{\frac{1}{2}(n+\kappa)}$	$2^{\frac{1}{2}(n+\kappa)}$	$2^{\frac{1}{2}(n+\kappa)}$
[JLMMN17]	GXHX	1	$2^{\frac{1}{2}(n+\kappa)}$	none	$2^{\frac{1}{2}(n+\kappa)}$
[Mennink15]	$\tilde{F}[2]$	1	2^n	2^n	N.A.
[LisRivWag11]	LRW1	2	$2^{n/2}$	$2^{n/2}$	$2^{\frac{2}{3}(n+\kappa)}$
[LanShrTer12]	CLRW2	2	$2^{3n/4}$	$2^{3n/4}$	$2^{\frac{2}{3}(n+\kappa)}$
[LeeLee18]	XHX2	2	$2^{\frac{2}{3}(n+\kappa)}$	none	$2^{\frac{2}{3}(n+\kappa)}$

Take-Aways

- Cryptanalysis of the generalized tweakable FX construction for $r \geq 1$ rounds in $\mathcal{O}(2^{\frac{r}{r+1}(n+\kappa)})$ query complexity under standard assumptions.
- Shows tightness of proofs of GXHX and XHX2 which in turn show it is information theoretically optimal for $r = 1, 2$ rounds.
- Gives a security upper-bound for this strategy with $r \geq 3$ rounds.

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Open Questions:

- How simple can the subkey functions be while maintaining security?
- Can we prove security for $r \geq 3$ rounds?
- What concrete application for those improved schemes?