A Polynomial Spilling Heuristic: Layered Allocation

Boubacar Diouf[1 2], Albert Cohen[1 2], Fabrice Rastello[1 2]

[1]INRIA

[2]École Normale Supérieure de Paris

[3]École Normale Supérieure de Lyon

Register Allocation

The register allocation problem maps temporary variables to machine registers

The Allocation/Spilling Problem

- The allocation chooses the register residents
- It also aims at minimizing the load/store overhead

Assignment/Coloring

- The coloring decides which register is used by which variable

Decoupling

- For the moment, let us assume that these two problems can be decoupled

Register Allocation

The register allocation problem maps temporary variables to machine registers

The Allocation/Spilling Problem

- The allocation chooses the register residents
- It also aims at minimizing the load/store overhead

Assignment/Coloring

- The coloring decides which register is used by which variable

Decoupling

- For the moment, let us assume that these two problems can be decoupled

Register Allocation

The register allocation problem maps temporary variables to machine registers

The Allocation/Spilling Problem

- The allocation chooses the register residents
- It also aims at minimizing the load/store overhead

Assignment/Coloring

- The coloring decides which register is used by which variable

Decoupling

- For the moment, let us assume that these two problems can be decoupled

Register Allocation

The register allocation problem maps temporary variables to machine registers

The Allocation/Spilling Problem

- The allocation chooses the register residents
- It also aims at minimizing the load/store overhead

Assignment/Coloring

- The coloring decides which register is used by which variable

Decoupling

- For the moment, let us assume that these two problems can be decoupled

A bit of Terminology

- **Maxlive**: the maximum number of simultaneously live variables
- Given V a set of variables of a program and R a number of available registers

Two sub-problems

- The **lowering problem** finds S, a subset of V, of minimum cost to spill in order to decrease maxlive by a small number
- The **single layer allocation problem** finds A, a subset of V, of maximum cost to allocate to a small number of registers

Two Approaches to the Allocation Problem

- The **layered allocation** incrementally solves the single layer allocation problem until the sum of the used registers reaches R
- The **incremental lowering** incrementally solves the lowering problem until maxlive reaches R

A bit of Terminology

- **Maxlive**: the maximum number of simultaneously live variables
- Given V a set of variables of a program and R a number of available registers

Two sub-problems

- The **lowering problem** finds S, a subset of V, of minimum cost to spill in order to decrease maxlive by a small number
- The **single layer allocation problem** finds A, a subset of V, of maximum cost to allocate to a small number of registers

Two Approaches to the Allocation Problem

- The **layered allocation** incrementally solves the single layer allocation problem until the sum of the used registers reaches R
- The **incremental lowering** incrementally solves the lowering problem until maxlive reaches R

A bit of Terminology

- **Maxlive**: the maximum number of simultaneously live variables
- Given V a set of variables of a program and R a number of available registers

Two sub-problems

- The **lowering problem** finds S, a subset of V, of minimum cost to spill in order to decrease maxlive by a small number
- The **single layer allocation problem** finds A, a subset of V, of maximum cost to allocate to a small number of registers

Two Approaches to the Allocation Problem

- The **layered allocation** incrementally solves the single layer allocation problem until the sum of the used registers reaches R
- The **incremental lowering** incrementally solves the lowering problem until maxlive reaches R

A bit of Terminology

- **Maxlive**: the maximum number of simultaneously live variables
- Given V a set of variables of a program and R a number of available registers

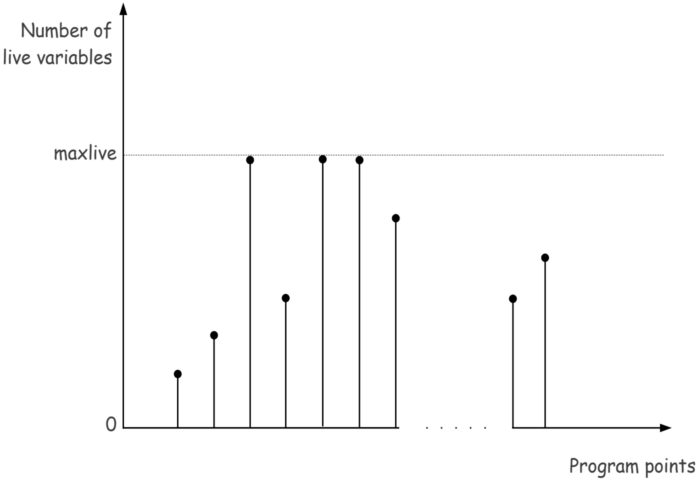
Two sub-problems

- The **lowering problem** finds S, a subset of V, of minimum cost to spill in order to decrease maxlive by a small number
- The **single layer allocation problem** finds A, a subset of V, of maximum cost to allocate to a small number of registers

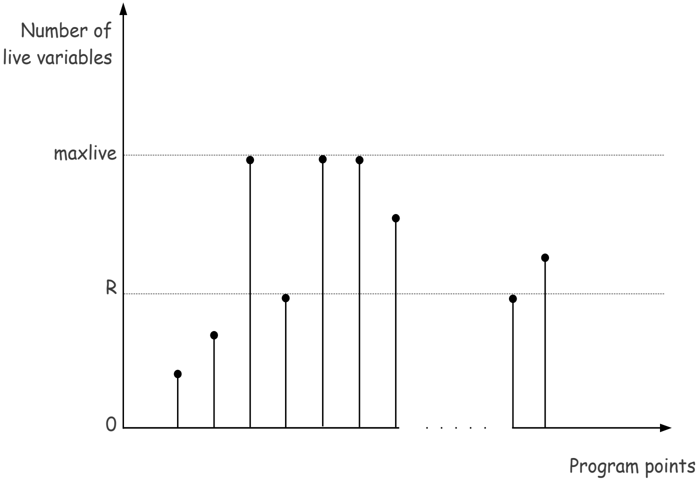
Two Approaches to the Allocation Problem

- The **layered allocation** incrementally solves the single layer allocation problem until the sum of the used registers reaches R
- The **incremental lowering** incrementally solves the lowering problem until maxlive reaches R

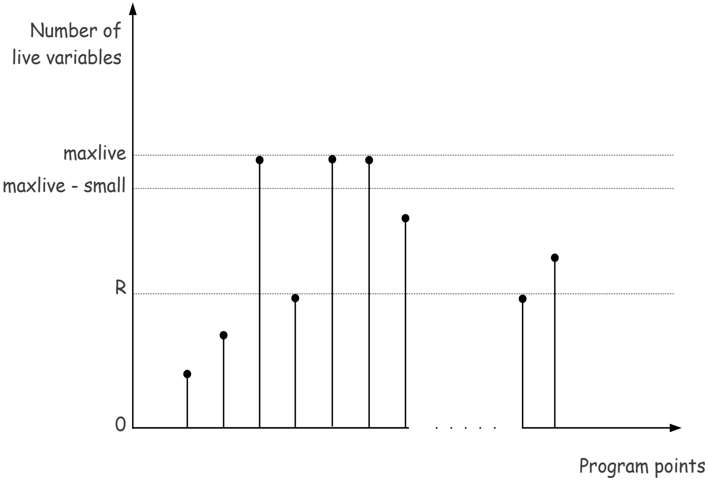
Lowering vs. Layering



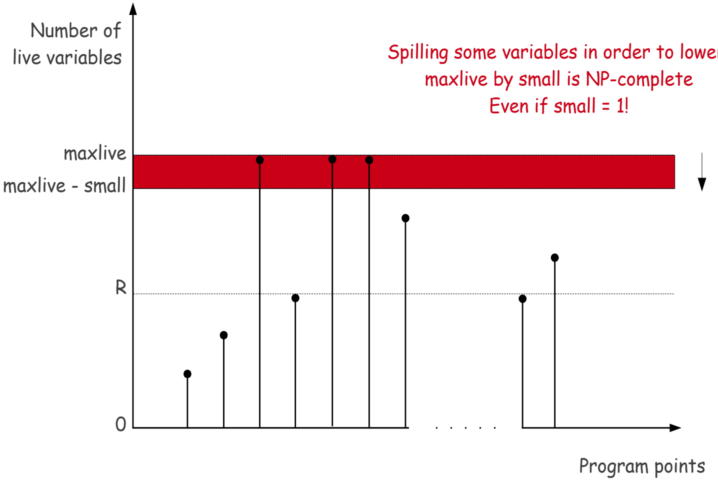
Lowering vs. Layering



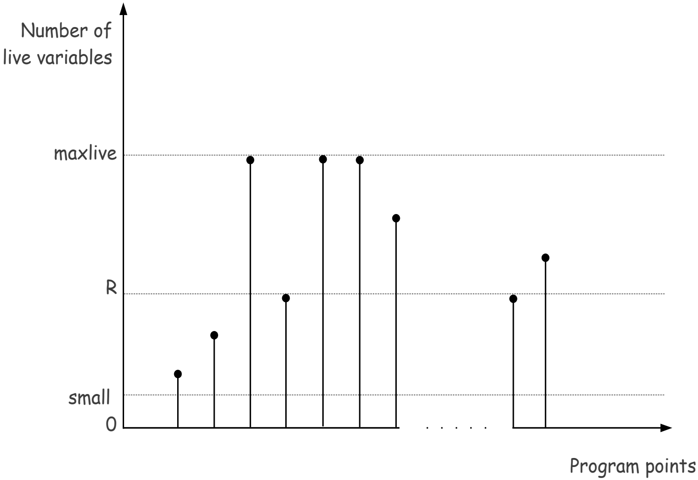
Lowering vs. Layering



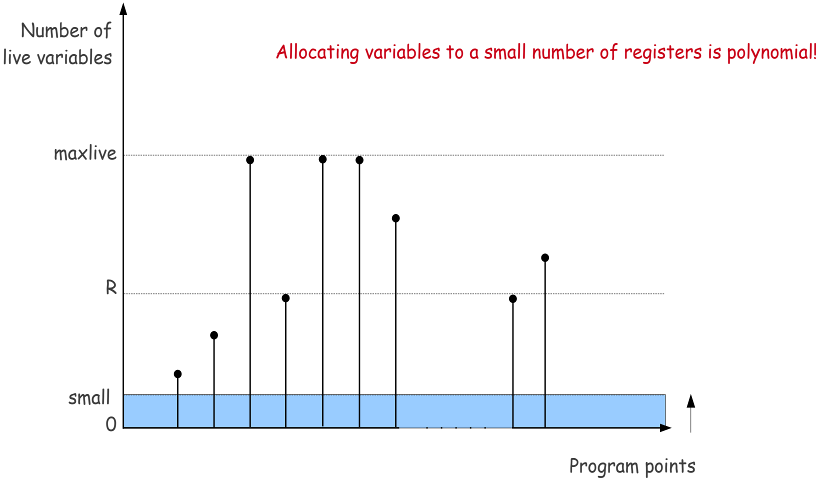
Lowering vs. Layering



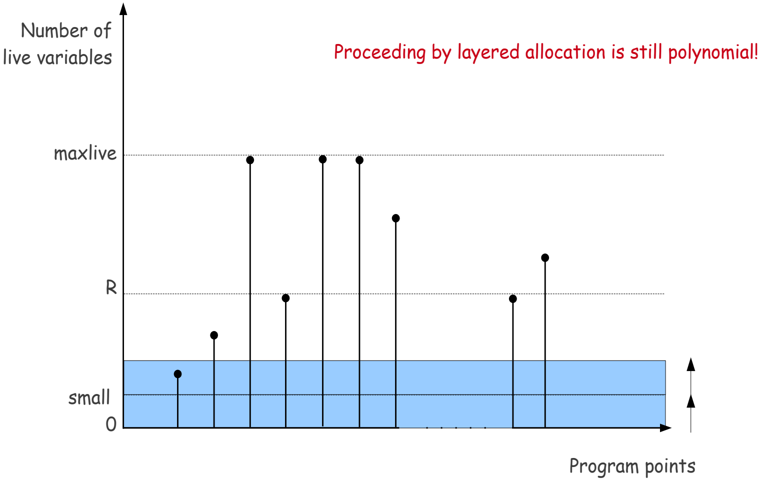
Lowering vs. Layering



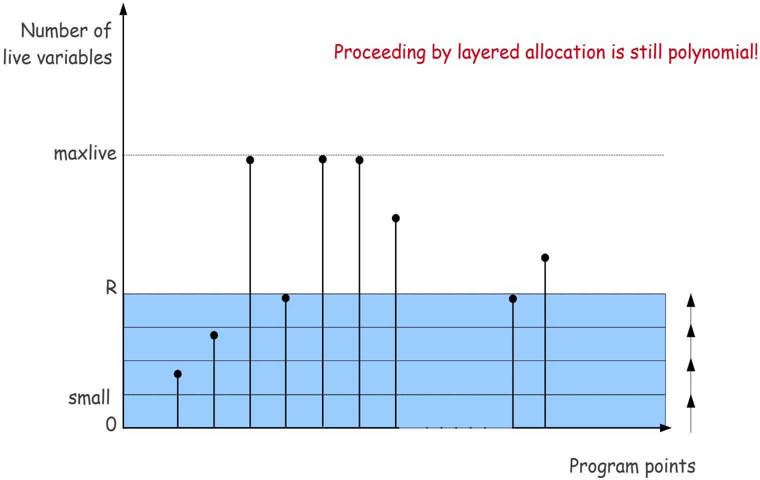
Lowering vs. Layering



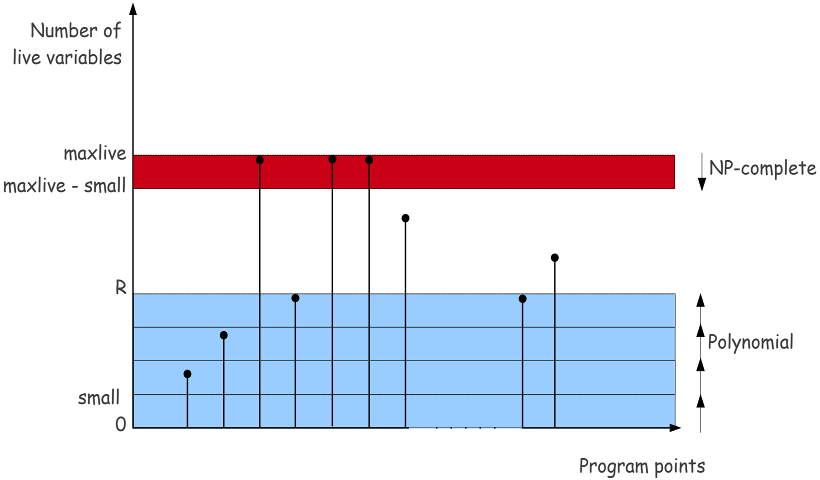
Lowering vs. Layering



Lowering vs. Layering



Lowering vs. Layering



Why should the Layered Allocation be Close to Optimal?

- Let us assume that we have a program P
- When $R+1$ registers are available, let us call $SPILL^P_{R+1}$ the optimal set of variables to spill to make a coloring possible
- For most programs, $SPILL^P_{R+1} \subset SPILL^P_R$ **[Diouf'10]**
- Hence, for most programs, $ALLOC^P_R \subset ALLOC^P_{R+1}$

Taxonomy of the Approaches

Approach	Complexity	Quality
Allocation/Spilling	NP-complete	Optimal
Layered Allocation	Polynomial	Close to optimal
Incremental lowering-optimal	NP-complete	???
Incremental lowering-heuristic	Polynomial	Not-optimal

To make it clear!

- The Allocation problem is NP-complete
- The Layered allocation is a heuristic that is close to optimal allocation
- We are not turning an NP-complete problem into a polynomial one

Taxonomy of the Approaches

Approach	Complexity	Quality
Allocation/Spilling	NP-complete	Optimal
Layered Allocation	Polynomial	Close to optimal
Incremental lowering-optimal	NP-complete	???
Incremental lowering-heuristic	Polynomial	Not-optimal

To make it clear!

- The Allocation problem is NP-complete
- The Layered allocation is a heuristic that is close to optimal allocation
- We are not turning an NP-complete problem into a polynomial one

Taxonomy of the Approaches

Approach	Complexity	Quality
Allocation/Spilling	NP-complete	Optimal
Layered Allocation	Polynomial	Close to optimal
Incremental lowering-optimal	NP-complete	???
Incremental lowering-heuristic	Polynomial	Not-optimal

To make it clear!

- The Allocation problem is NP-complete
- The Layered allocation is a heuristic that is close to optimal allocation
- We are not turning an NP-complete problem into a polynomial one

Taxonomy of the Approaches

Approach	Complexity	Quality
Allocation/Spilling	NP-complete	Optimal
Layered Allocation	Polynomial	Close to optimal
Incremental lowering-optimal	NP-complete	???
Incremental lowering-heuristic	Polynomial	Not-optimal

To make it clear!

- The Allocation problem is NP-complete
- The Layered allocation is a heuristic that is close to optimal allocation
- We are not turning an NP-complete problem into a polynomial one

Outline

Introduction

Layered Approach
- Layered-Heuristic Allocation: General Graphs
- Layered-Optimal Allocation: Chordal Graphs

Experimental Evaluation

Conclusion

Two Heuristics for the Spilling Problem

Input:

1. A register allocation problem where each variable has an estimated spill cost
2. A number of available registers

Objective:

We want to perform an allocation that minimizes the cost of all the spilled variables

Two graph-based solutions :

- The general approach: Layered-Heuristic Register Allocator
- The SSA-based approach: Layered-Optimal Register Allocator

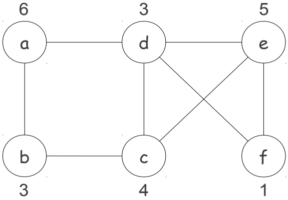
The General Approach

Given an interference graph of a program and `R` available registers (colors)

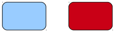
1. Assume that we have `one` register
2. We approximate the set of nodes of maximum cost/weight to allocate with *one* register: a layer. This layer is an independent set.
3. Remove the nodes of the layer from the graph at the next iteration

Repeat these instructions until we reach `R` or we allocate all the variables

How the Layered-Heuristic Works

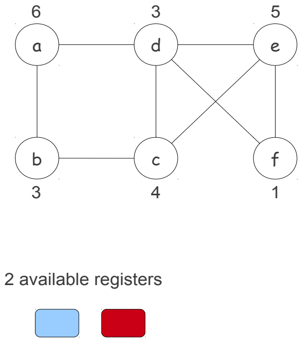


2 available registers



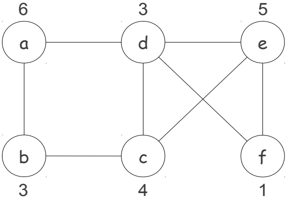
How the Layered-Heuristic Works

Variables sorted by decreasing cost: a, e, c, b, d, f



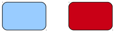
How the Layered-Heuristic Works

Variables sorted by decreasing cost: a, e, c, b, d, f



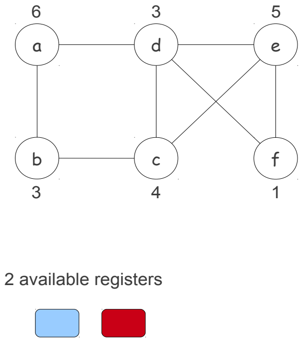
I-Set-1: {a

2 available registers



How the Layered-Heuristic Works

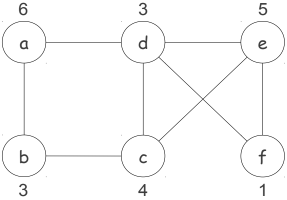
Variables sorted by decreasing cost: e, c, b, d, f



I-Set-1: {a,e}

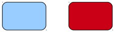
How the Layered-Heuristic Works

Variables sorted by decreasing cost: c, b, d, f



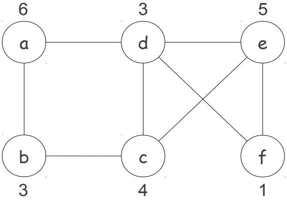
I-Set-1: {a,e}

2 available registers



How the Layered-Heuristic Works

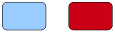
Variables sorted by decreasing cost: b, d, f



I-Set-1: {a,e}

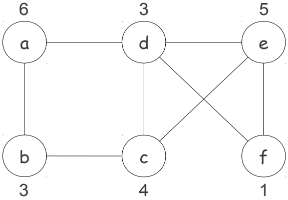
I-Set-2: {c,

2 available registers



How the Layered-Heuristic Works

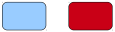
Variables sorted by decreasing cost: b, d



I-Set-1: {a,e}

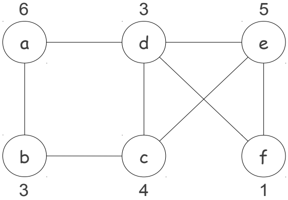
I-Set-2: {c,f}

2 available registers



How the Layered-Heuristic Works

Variables sorted by decreasing cost:

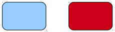


I-Set-1: {a,e}

I-Set-2: {c,f}

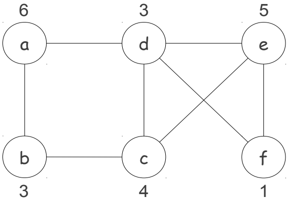
I-Set-3: {b,d}

2 available registers



How the Layered-Heuristic Works

Variables sorted by decreasing cost:



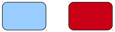
I-Set-1: {a,e}

I-Set-2: {c,f}

I-Set-3: {b,d}

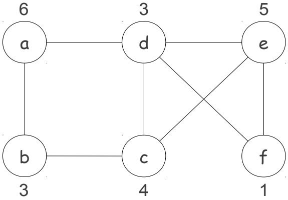
I-Sets sorted by decreasing cost: I-Set-1, I-Set-3, I-Set-2

2 available registers



How the Layered-Heuristic Works

Variables sorted by decreasing cost:



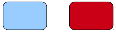
I-Set-1: {a,e}

I-Set-2: {c,f}

I-Set-3: {b,d}

I-Sets sorted by decreasing cost: I-Set-1, I-Set-3, I-Set-2

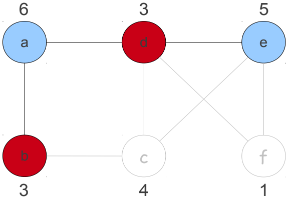
2 available registers



The cost of the allocation is **5**

How the Layered-Heuristic Works

Variables sorted by decreasing cost:



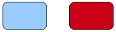
I-Set-1: {a,e}

I-Set-2: {c,f}

I-Set-3: {b,d}

I-Sets sorted by decreasing cost: I-Set-1, I-Set-3, I-Set-2

2 available registers



The cost of the allocation is **5**

Outline

Introduction

Layered Approach

Layered-Heuristic Allocation: General Graphs

Layered-Optimal Allocation: Chordal Graphs

Experimental Evaluation

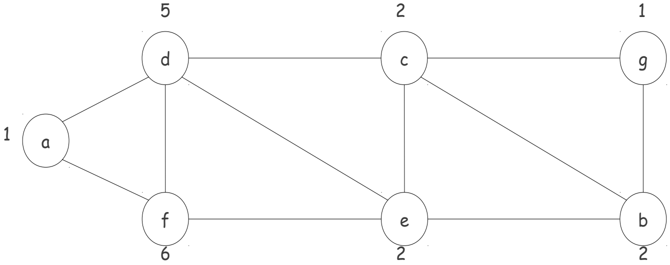
Conclusion

SSA-based Interference Graphs

The interference graph of an SSA-based program is chordal

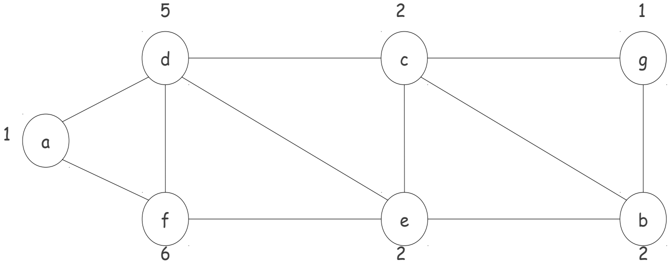
1. The allocation problem can be decoupled from the coloring problem thanks to maxlive
2. Hence, the maximum weighted independent set can be found optimally **[Frank'75]**

The Maximum Weighted Independent Set Algorithm



Weighted graph

The Maximum Weighted Independent Set Algorithm

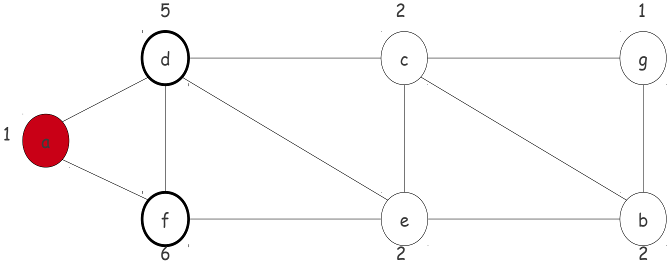


Weighted graph

iteration	a	f	d	e	b	g	c	red vertices
-	1	6	5	2	2	1	2	Ø

Red vertices

The Maximum Weighted Independent Set Algorithm

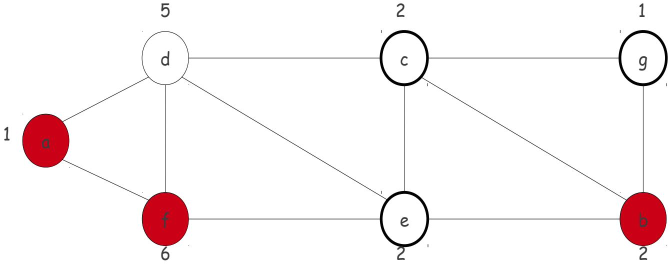


Weighted graph

iteration	a	f	d	e	b	g	c	red vertices
-	1	6	5	2	2	1	2	∅
1		5	4	2	2	1	2	a

Red vertices

The Maximum Weighted Independent Set Algorithm

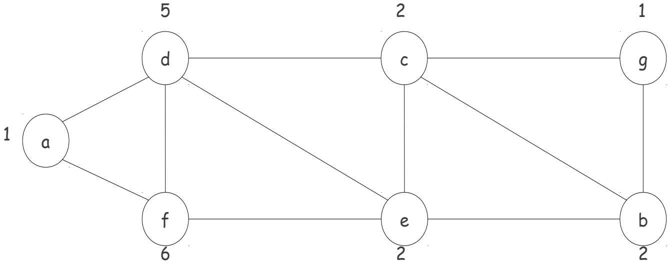


Weighted graph

iteration	a f d e b g c							red vertices
-	1	6	5	2	2	1	2	Ø
1		5	4	2	2	1	2	a
2			*-1*	*-3*	2	1	2	f, a
5						*-1*	0	b,f,a

Red vertices

The Maximum Weighted Independent Set Algorithm



Weighted graph

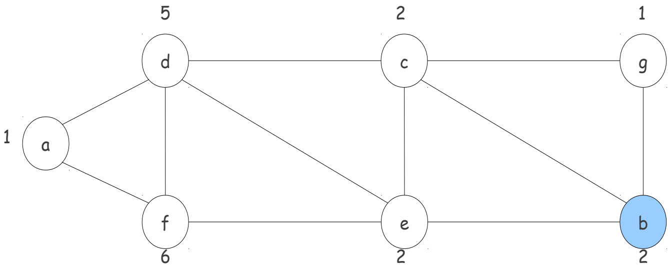
iteration	a	f	d	e	b	g	c	red vertices
-	1	6	5	2	2	1	2	Ø
1		5	4	2	2	1	2	a
2			*-1*	*-3*	2	1	2	f, a
5						*-1*	0	b,f,a

Red vertices

iteration	red vertices	blue vertices
-	b,f,a	Ø

Blue vertices

The Maximum Weighted Independent Set Algorithm



Weighted graph

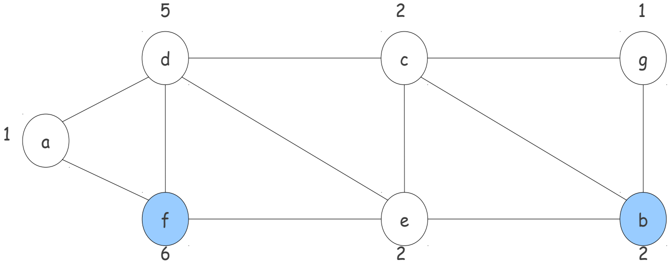
iteration	a	f	d	e	b	g	c	red vertices
-	1	6	5	2	2	1	2	Ø
1		5	4	2	2	1	2	a
2			*-1*	*-3*	2	1	2	f, a
5						*-1*	0	b,f,a

Red vertices

iteration	red vertices	blue vertices
-	b,f,a	Ø
1	f,a	b

Blue vertices

The Maximum Weighted Independent Set Algorithm



Weighted graph

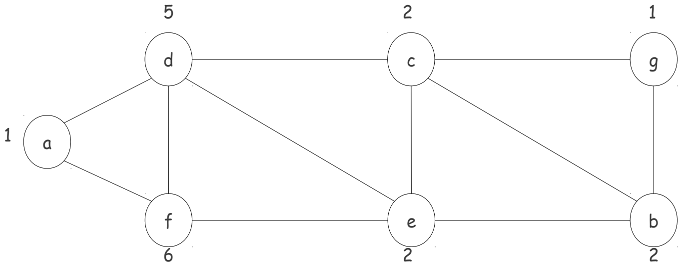
iteration	a	f	d	e	b	g	c	red vertices
-	1	6	5	2	2	1	2	Ø
1		5	4	2	2	1	2	a
2			*-1*	*-3*	2	1	2	f, a
5						*-1*	0	b,f,a

Red vertices

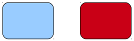
iteration	red vertices	blue vertices
-	b,f,a	Ø
1	f,a	b
2	Ø	b, f

Blue vertices

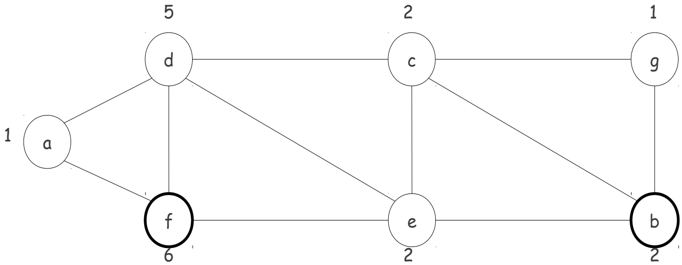
How the Layered-Optimal Allocator Works



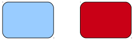
2 available registers



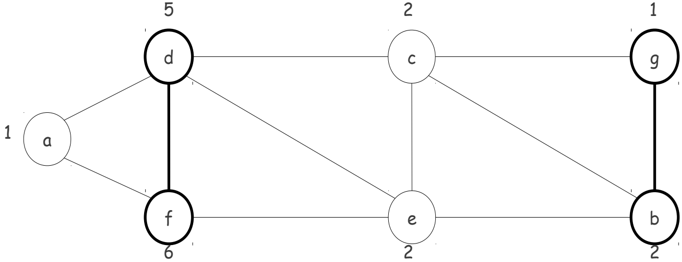
How the Layered-Optimal Allocator Works



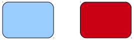
2 available registers



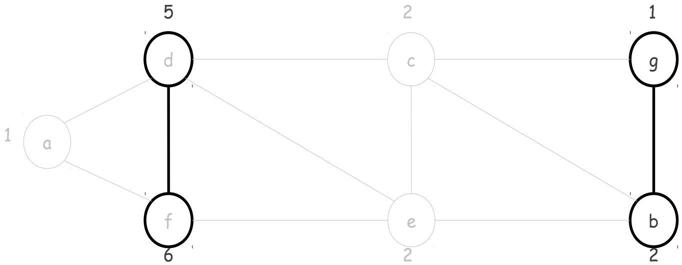
How the Layered-Optimal Allocator Works



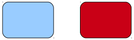
2 available registers



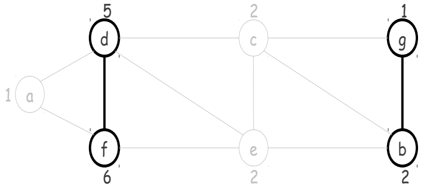
How the Layered-Optimal Allocator Works



2 available registers

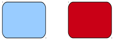


A First Improvement: Weights Bias

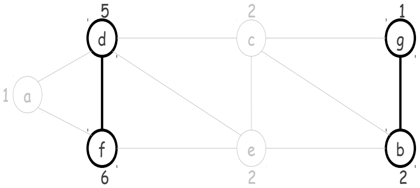


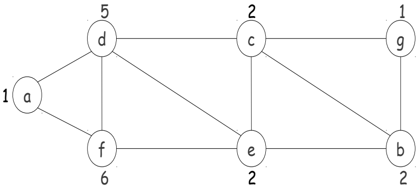
Allocated variables: {f, b, d, g}
Allocation-Cost = 14

2 available registers



A First Improvement: Weights Bias

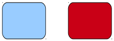




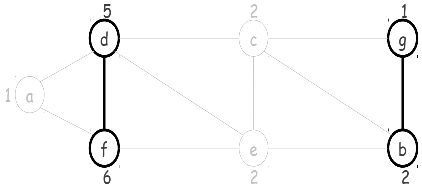
Allocated variables: {f, b, d, g}
Allocation-Cost = 14

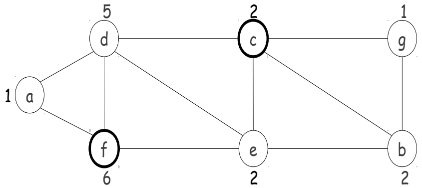
Allocated variables: { }

2 available registers



A First Improvement: Weights Bias



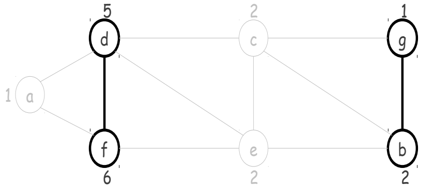


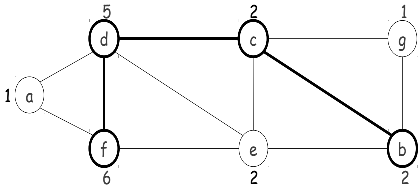
Allocated variables: {f, b, d, g}
Allocation-Cost = 14

Allocated variables: {f, c}

2 available registers

A First Improvement: Weights Bias

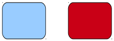




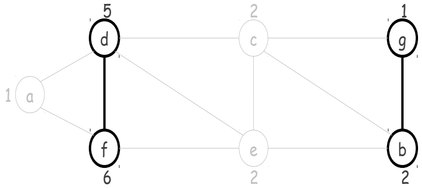
Allocated variables: {f, b, d, g}
Allocation-Cost = 14

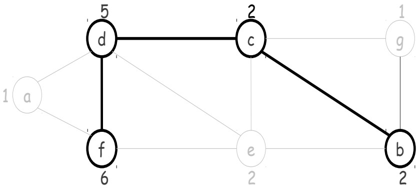
Allocated variables: {f, c, d, b}

2 available registers



A First Improvement: Weights Bias

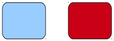




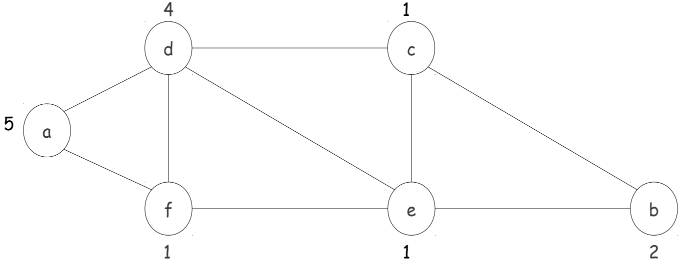
Allocated variables: {f, b, d, g}
Allocation-Cost = 14

Allocated variables: {f, c, d, b}
Allocation-Cost = 15

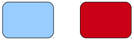
2 available registers



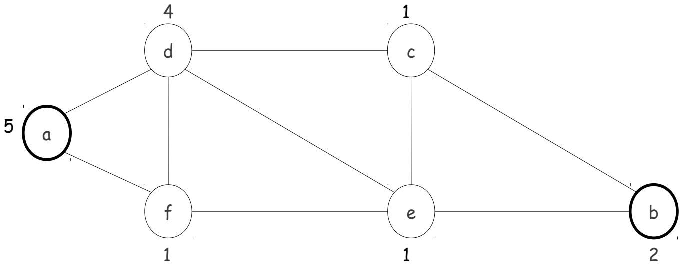
A Second Improvement: A Fixed Point Iteration



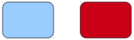
2 available registers



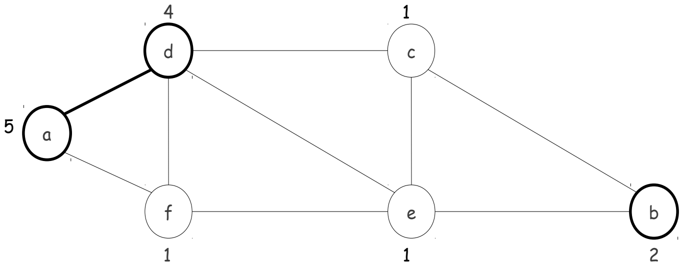
A Second Improvement: A Fixed Point Iteration



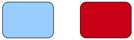
2 available registers



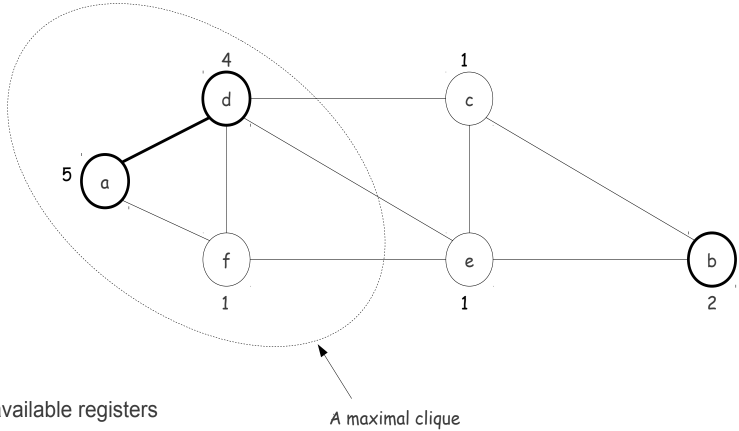
A Second Improvement: A Fixed Point Iteration



2 available registers

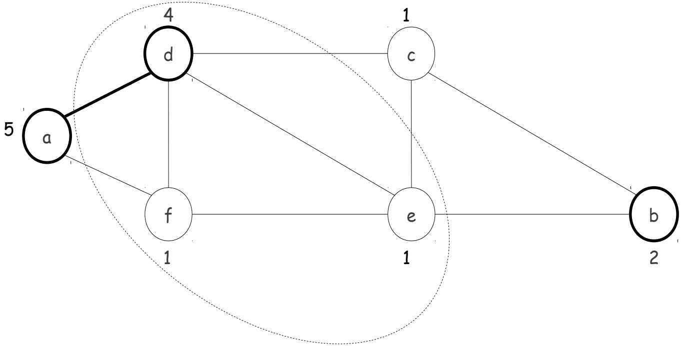


A Second Improvement: A Fixed Point Iteration

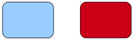


2 available registers

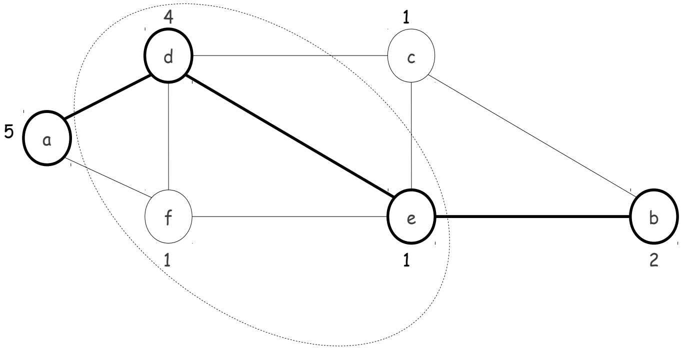
A Second Improvement: A Fixed Point Iteration



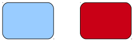
2 available registers



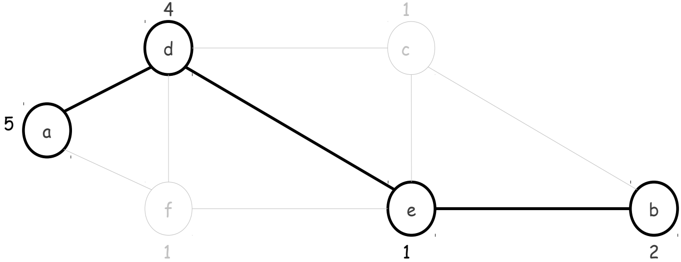
A Second Improvement: A Fixed Point Iteration



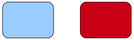
2 available registers



A Second Improvement: A Fixed Point Iteration



2 available registers



Outline

Introduction

Layered Approach

Layered-Heuristic Allocation: General Graphs

Layered-Optimal Allocation: Chordal Graphs

Experimental Evaluation

Conclusion

Evaluating the Layered-Heuristic Allocator

Architectures

- x86

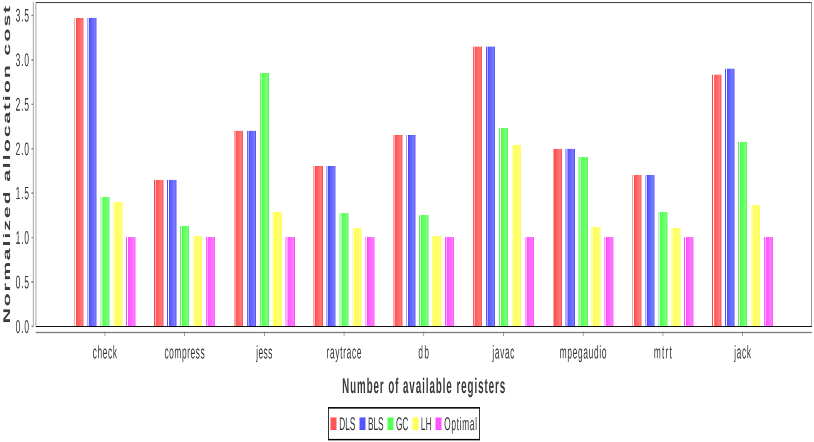
Benchmarks extracted from JikesRVM

- SPEC JVM 98

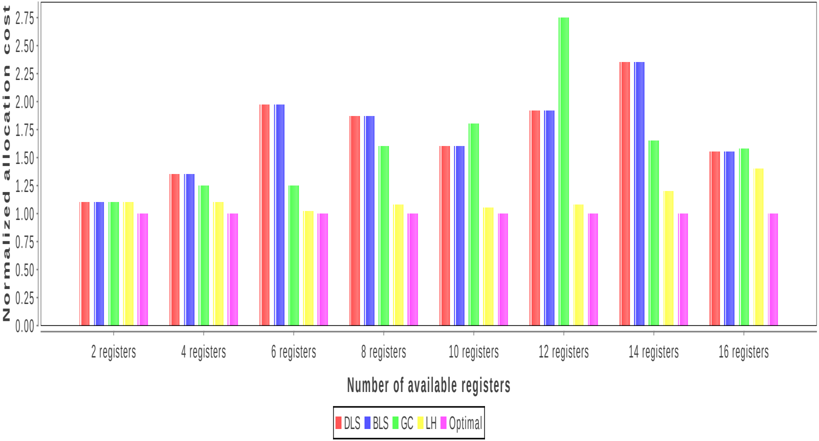
Algorithms

- LS: the linear scan implemented in JikesRVM
- BLS: a variant of the Belady's furthest -first
- GC: the Chaitin-Briggs optimistic graph coloring
- Optimal: an ILP-based Allocator
- LH: Layered Heuristic

Evaluating the Layered-Heuristic Allocator



Evaluating the Layered-Heuristic Allocator



Evaluating the Layered-Optimal Allocator

Architectures

- ARMv7
- ST231

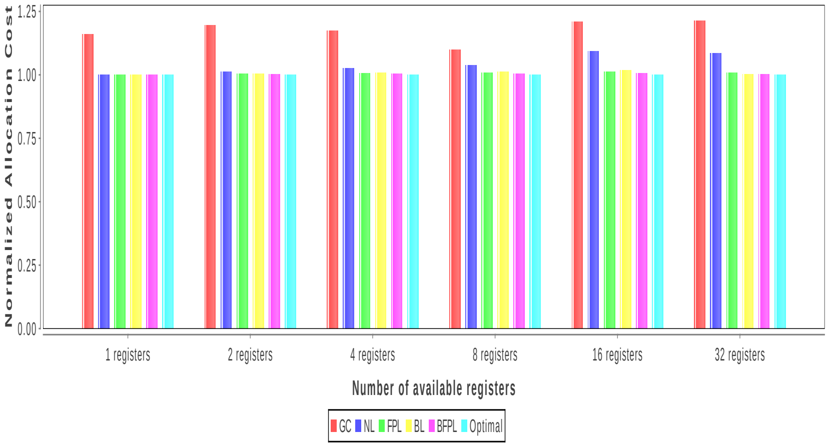
Benchmarks

- eembc
- lao-kernels
- SPEC CPU 2000int

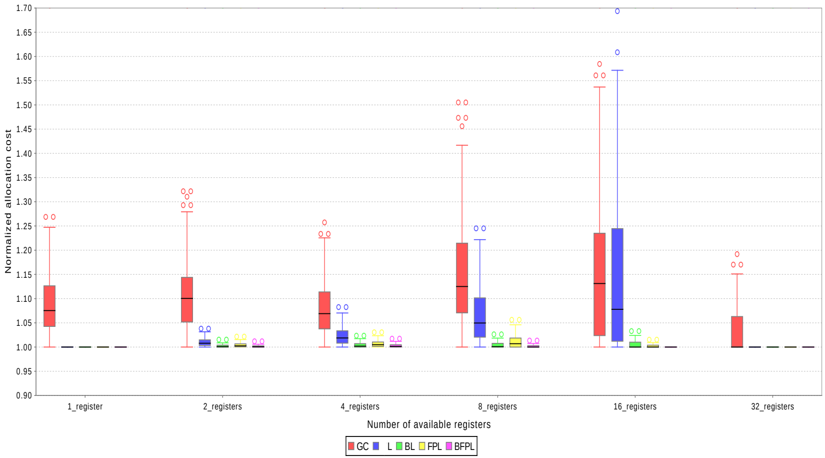
Algorithms

- GC: the Chaitin-Briggs optimistic graph coloring
- Optimal: an ILP-based Allocator
- L: our baseline Layered-Optimal approach
- BL: the biased variant of our Layered-Optimal
- FPL: the fixed-point variant of our Layered-Optimal
- BFPL: the biased and fixed-point variant of our Layered-Optimal

Evaluating the Layered-Optimal Allocator



Evaluating the Layered-Optimal Allocator



Outline

Introduction

Layered Approach

Layered-Heuristic Allocation: General Graphs

Layered-Optimal Allocation: Chordal Graphs

Experimental Evaluation

Conclusion

Conclusion

Contributions

- Layered allocation: polynomial and close to optimal allocation
- Iteratively allocate instead of (classical) iteratively spilling
- The approach works on general graphs and on SSA-based graphs

Why a Graph-based Approach

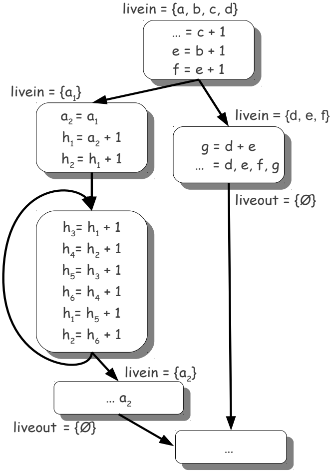
Approach	**Graph-based**	**Program-based**
UsefulNess	Not easy	Easy
Profitability	Easy	Difficult

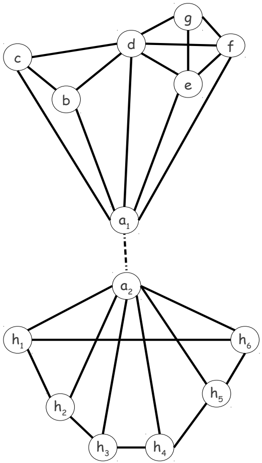
Why a Graph-based Approach

Approach	**Graph-based**	**Program-based**
UsefulNess	Not easy	Easy
Profitability	Easy	Difficult

Why a Graph-based Approach

Assuming we have three available registers



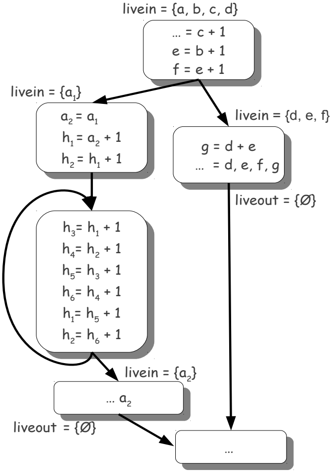


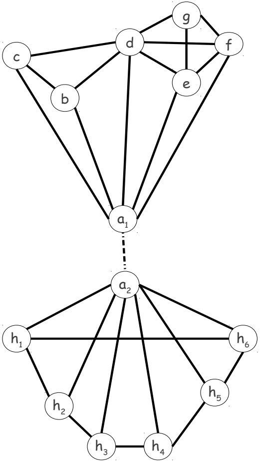
Why a Graph-based Approach

Approach	**Graph-based**	**Program-based**
UsefulNess	Not easy	Easy
Profitability	Easy	Difficult

Why a Graph-based Approach

Assuming we have three available registers





Why a Graph-based Approach

Approach	**Graph-based**	**Program-based**
UsefulNess	Not easy	Easy
Profitability	Easy	Difficult

Why a Spill Everywhere Problem?

1. A solution of a spill everywhere problem can play the role of an oracle
2. The cost of the store favors spilling entire live range instead of two sub-ranges of different variables
3. The queuing mechanism is highly sensitive to the number of simultaneously spilled variables
4. The case where a store is considered to have no cost is equivalent to a spill everywhere formulation