Drawing Diagrams From Labelled Graphs

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1 Introduction

The main idea of Euler Diagrams is to propose a visual representation of the non empty set intersections of a collection of sets which maintains the connectivity of each set and such that each zone corresponding to a given sets intersection appears only once. Additional drawing conditions are usually introduced in the literature leading to the following list of conditions :
-1- Contours should be simple curves, so that contours that cross themselves are not allowed
-2- Disconnected zones may not be allowed, so that zones cannot appear more than once in a diagram
-3- Triple points may not be allowed, so that only two contours can intersect at any given point
-4- Concurrent contours may not be allowed, so a line segment cannot represent the border of 2 or more contours.
-5- The shape of contours may be restricted to certain shapes such: as circular, oval, rectangular or convex shapes.
Extended Euler diagrams introduced in [4] must satisfy only the two first conditions. The well-formedness conditions introduced in [2] include conditions 1 to 4.
By duality, Euler diagrams can be associated to the drawing of planar labelled graphs, where each region corresponds to a vertex and the adjacency of zones are represented by the edges. In [4], we have shown by a constructive method that there exists an extended Euler diagram representation for any collection of $n < 9$ sets. In this paper, we present a method building a diagram from the drawing of a planar labelled graph and discuss our results. Most of the labelled graphs used in this paper have been built following the method of [4,5].

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2 Definitions

Let us introduce first the definitions of extended Euler diagrams and L_connected labelled graphs.

Definition 1 *Let L be a finite set of labels and C a set of simple closed (Jordan) curves in the plane. We say that C is* labelled *by L when each curve c of C is associated with a couple $(\lambda(c), sign(c))$ where $\lambda(c) \in L$ and $sign(c) \in \{+, -\}$.*
To each labelled curve c of C corresponds a zone $\zeta(c)$ defined by:
- if $sign(c) = +$, then $\zeta(c) = int(c)$
- if $sign(c) = -$, then $\zeta(c) = ext(c)$
We note $\overline{\zeta(c)}$ the complement region of $\zeta(c)$ in the plane.

Definition 2 *Let C be a set of simple closed curves of the plane labelled by L. Given any subset R of L, let $region(R)$ be:*

$$region(R) = \begin{cases} \bigcap_{\lambda(c)\in R} \zeta(c), & \text{if } R \neq \emptyset \\ \bigcap_{c\in C} \overline{\zeta(c)}, & \text{if } R = \emptyset \end{cases}$$

The regions of C are :

$$regions(C) = \{R | R \subseteq L \text{ and } region(R) \text{ is non empty } \}$$

Definition 3 *An* extended Euler diagram *for $\mathcal{L} \subseteq \mathcal{P}(L)$ is a set C of planar simple closed curves labelled by L such that:*

(i) L is a finite set of labels

(ii) C is a set of Jordan curves labelled by L and verifying:
(a) $\forall l \in L, \exists c \in C$, $\lambda(c) = l$ and $sign(c) = +$.
(b) if $\lambda(c) = \lambda(c')$, $c \neq c'$ and $sign(c) = sign(c')$ then c and c' do not intersect
(c) if $\lambda(c) = \lambda(c')$, $c \neq c'$ and $sign(c) = +$, then $sign(c') = -$ and $c' \subset int(c)$

(iii) when $R \neq \emptyset$, $region(R)$ is connected.

The set of extended Euler diagrams is noted $\mathcal{EED}$.

Definition 4 *A* labelled graph *is a triple $G(L, V, E)$ where:*

(i) L is a finite set of labels

(ii) V is a set of labelled vertices, *i.e.:*
(a) each vertex v is labelled with a set of labels $l(v) \subseteq L$
(b) two distinct vertices v and w of V have distinct sets of labels.

(iii) E is a set of edges such that:
(a) each edge $e = (v, w)$ of E is labelled with a set of labels $l(e) = l(v) \cap l(w)$
(b) if $e \in E$ then $l(e) \neq \emptyset$

Definition 5 *Let $G(L, V, E)$ be a labelled graph. We say that $G(L, V, E)$ is* L_connected *if and only if for all l in L, the subgraph G' of $G(L, V, E)$ on the set V' of vertices of V having l in its set of labels is connected.*

Definition 6 *Let C be an extended Euler diagram on L, the dual $G(L, V, E)$ of C is the L_connected*

labelled graph defined by:
- each non empty subset R of L such that $region(R)$ is non empty is associated to a vertex v of V with $l(v) = R$,
- when two non empty subsets R and R' of L are such that $region(R)$ and $region(R')$ are adjacent, then E contains a vertex e joining the two corresponding vertices and $l(e) = R \cap R'$.

3 From planar L_connected labelled graphs to Euler diagrams

Let $D(G)$ be a straight-line planar drawing of a L_connected labelled graph $G(L, V, E)$. The following process builds an extended Euler diagram C on L such that $G(L, V, E)$ is its dual graph.

-1- We temporary remove the dangling edges from each internal face of $D(G)$.

-2- Each internal face F of $D(G)$ which is not triangular is triangulated.
We now have a triangulation $F_1, ..., F_n$ representing $G(L, V, E)$.

-3- If an internal triangular face $F_i = (v_1, v_2, v_3)$ contains at least a dangling edge $v'v_1$ connected to v_1, F_i is subdivided in three triangles, by the introduction of two new edges connecting v' to v_2 and v_3.
Then we obtain a drawing of a graph formed by triangular faces connected by edges and which can contain "tree-like groups of edges" in the external border of the graph.

-4- Each vertex v of G is associated to a planar region $region(l(v))$.
Each triangular face $F_i = (v_i, v'_i, v"_i)$ is subdivided in three subregions as follows :

- The centroid w_i of F_i is computed and three line segments joining w_i to the middle of the three edges of F_i are formed. These line segments will be a part of the boundaries of the regions associated respectively to v_i, v'_i and $v"_i$.
- When an edge $e = (v, v')$ is on the boundary of $D(G)$, two subregions associated to its two extremities are formed. Three points external to $D(G)$ are computed:
  - p_m belongs to the perpendicular bisector of e
  - p_v and $p_{v'}$ belong to the bisectors of e and the segments adjacent to e in the external face of $D(G)$.

  Then three line segments (p_m, p_v), $(p_m, p_{v'})$ and (p_m, p_e), where p_e is the middle of e (cf. Figure 1) are built.

-5- Then, to draw the Euler diagram, we use parallel lines to draw the contours on the common parts of their boundaries.
The contour line associated to the label l will cut the edge $e = (v, v')$ iff $l \in l(v) \cup l(v')$ and $l(v) \cap l(v')$ does not contain l. Let suppose that L' contains the labels whose contour lines cut e. We order the labels of L' as follows:
- L'_+ contains the labels belonging to $l(v)$ and not to $l(v')$ and its labels are ordered according to the order induced by L. $L_+ = \{l'_1, ..., l'_k\}$
- L'_- contains the labels belonging to $l(v')$ and not to $l(v)$ and its labels are ordered according

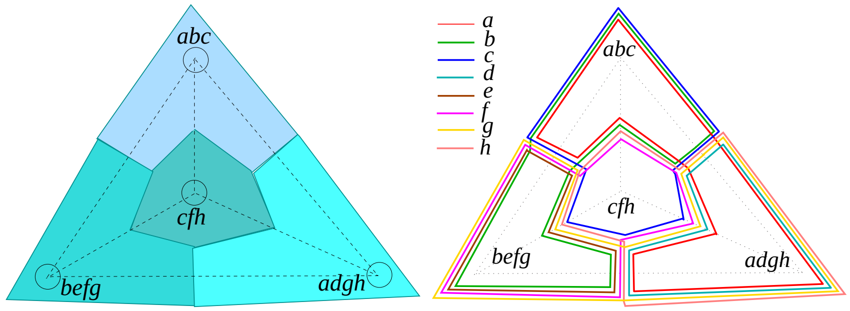

Fig. 1. Left: The regions corresponding to $D(G)$ with $V = \{abc, befg, adgh, cfh\}$ and $E = \{(abc, cfh), (abc, befg), (abc, adgh), (cfh, befg), (cfh, adgh)\}$. Right: The Euler diagram built from $D(G)$.

to the inverse order induced by L. $L_{-} = \{l'_{k1+1}, ..., l'_{k}\}$

4 Results and future works

Let us comment on some of our results.

- We have built extended Euler diagrams on the same collections of sets than in [2,3] for Figure 2 and by [1] for Figure 3. We see that the resulting diagrams are similar but in our case, the contour curves are unnecessarily stuck together and this affects the readability of the diagram.
  In Figure 2, the common portions of contour a and b may disappear, considering that the region

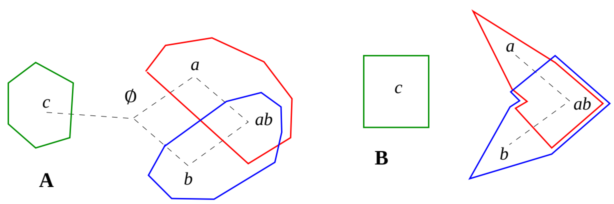

Fig. 2. A: the Euler diagram computed by [2]. B: the extended Euler diagram obtained by our method.

  $\{a, b\}$ is either adjacent to $\{a\}$ or adjacent to {b}. Thus contour b can be untied from contour a along the region $\{a, b\}$.
  In Figure 3, the order of the contours a, b, c and d has to be reversed and all the contours can be untied.
- In Figure 4 the way the contours are drawn on the branches of the tree-like L_connected labelled graph generates common portions of contours which are just drawing artifacts and can be removed.
- Considering the diagram of figure 1, we see that when the number of sets associated to a region increases, the diagram is difficult to understand. In this case, as most of the contours surround two regions, we cannot deform the contour to render the diagram more readable. There is only one L_labelled graph associated to this collection of sets. Thus, in this case, we should test on users alternative drawings such as the one of Figure 5.

Thus, we must improve the layout of the contours of the diagram:

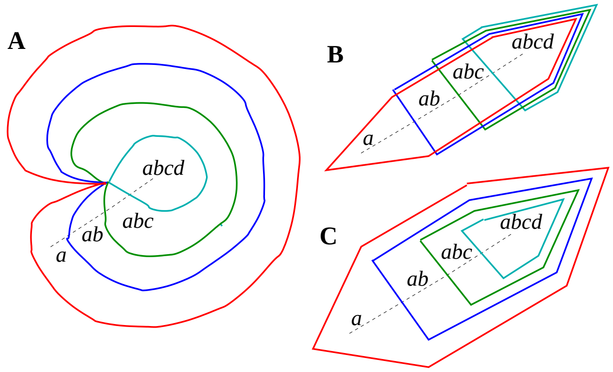

Fig. 3. A: the Euler diagram computed by [1]. B: the extended Euler diagram obtained by our method. C: the extended Euler diagram obtained after deformation of contours.

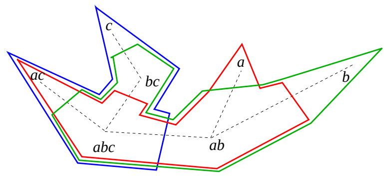

Fig. 4. An extended Euler diagrams built from a tree-like L_connected labelled graph

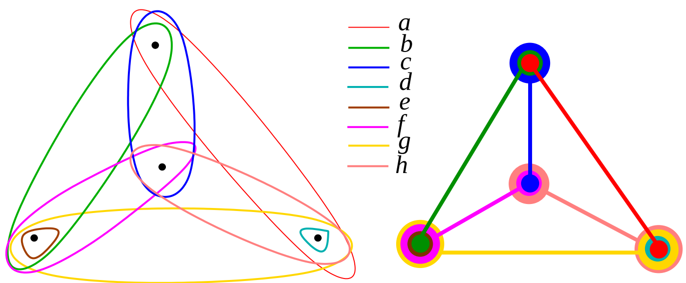

Fig. 5. Two alternative drawings

- by minimizing the number of common portions of curves in the drawing, modifying our drawing process,

- by modifying the choice of the L_connected labelled graph so that the drawing of the resulting diagram minimizes the number of common portions of curves.

- and finally, by using a smoothing method as in [3].

Nevertheless, when the diagrams are compact as in Figure 1, we must either consider alternative drawings or enhance the readability with appropriate interactive tools.

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