Computer Algebra, Symbolic Computation, and Automatic Control (CASCAC)

Proposal to the Parrot Program

Keywords: Mathematical systems theory, control theory, analysis and synthesis problems, nonlinear functional systems, functional equations, Ore algebras, symbolic computation, Gröbner bases, differential algebra, implementation, computer algebra systems.

1 Scientific context of the collaboration

Systems theory is a theory which asserts that organization can be found in the complex world and that such organization or "system" can be understood by means of concepts and principles which are independent from the particular domain studied (e.g., physics, engineering sciences, biology, economy). If the general laws governing this system can be discovered, then they can be used to analyze any system having similar features. Mathematical systems theory is a part of systems theory which aims at studying different classes of systems coming from engineering sciences (e.g., electrical, mechanical and chemical engineering), mathematical physics (e.g., elasticity, electromagnetism, hydrodynamics), biology, economy, communication... by means of common mathematical concepts, techniques, algorithms and softwares (e.g., discrete or continuous dynamical systems, linear or nonlinear, deterministic or stochastic, causal or acausal). An important issue of mathematical systems concerns the possibility to control systems by designing feedback laws. This theory can be traced back to Maxwell's work on governors but its modern development is mainly due to Kalman's work in the sixties [30].

Since Kalman's work (see [30] and the references therein), algebraic methods have been successfully applied to problems arising in mathematical systems theory (see, e.g., [29, 30, 39, 54, 48]). One of the reasons is that algebraic methods provide structural information about the system under consideration which cannot be obtained by using numerical methods alone. An algebraic approach to control systems leads to a classification of structural properties of the system (e.g., controllability, observability, flatness, invariants) [29, 30, 34, 39, 54, 48], which can be characterized in algebraic terms, checked by symbolic computation methods and implemented in computer algebra systems (e.g., `Maple` [41], `Mathematica` [42], `CoCoa` [20]). Moreover, synthesis problems can also be studied and solved within the algebraic approach (see, e.g., [29, 30, 39, 54, 48]).

Recently, within the algebraic approach of *skew polynomial rings* [43] of functional operators (e.g., differential, difference, shift operators), developed by Ore [47] (Noether's PhD student), many different classes of control systems have been treated simultaneously (e.g., continuous or discrete). The main advantage of studying systems in the language of skew polynomial rings is that properties of different classes of systems can be studied, characterized and checked by the same mathematical techniques, theorems and algorithms. In particular, algorithms for the study of systems can be stated and implemented in a way general enough for different types of systems to be covered at the same time.

This approach has recently been developed in two different and independent directions.

The first one, developed by Ülle Kotta's team at the Control Systems Department of the Institute of Cybernetics at TUT (Tallinn, Estonia), focusses on the study of nonlinear systems of functional equations defined by one functional operator (e.g., the differential operator for continuous systems, the shift operator for discrete systems). Using differential forms or Kähler differentials, nonlinear systems can be studied by means of matrices over a skew polynomial ring. Many important properties and problems of nonlinear control systems have been made constructive within this setting (e.g., realization, accessibility, observability, identifiability, linearization, reduction, transfer matrices, model matching). See [4, 5, 6, 33, 58] and the references therein. The corresponding algorithms have been implemented in the package `NLControl` [46] developed in the computer algebra system `Mathematica` [42]. A web interface has been developed for the package `NLControl` so that anyone could use it with only an internet browser (see [46, 58]). This web interface helped `NLControl` to become popular in the control theory community.

The second one, developed by Alban Quadrat (Inria Saclay - Île-de-France, Orsay, France) and his collaborators, focusses on the study of linear functional systems defined by more than one functional operators (e.g., differential time-delay systems, difference-differential systems, partial differential systems). This class of systems is called *multidimensional linear systems* in the control theory community. Multidimensional linear systems cannot be studied by simply applying elementary techniques on polynomial matrices as it is done for linear systems defined by one functional operator (see, e.g., [29, 30, 34, 39, 54, 48]). More sophisticated algebraic techniques must be used such as module theory and homological algebra [55]. For more information, see [13, 14, 17, 18, 19, 37, 44, 45, 50, 59]. Using *Gröbner basis techniques* [3, 11], important techniques of module theory and homological algebra were recently made constructive in [13, 17, 50, 51] for the important classes of *Ore algebras* (i.e., noncommutative multivariate polynomial algebras) [15] of functional operators useful in control theory. The main results have been implemented in the packages OreModules [14] and OreMorphisms [19] developed in the computer algebra system `Maple` [41].

2 Objectives of the collaboration

The CASCAC project is at the interface between computer algebra, symbolic computation and control theory.

The objective of the collaboration we want to develop within the PARROT program is to combine the two directions stated above and developed by the Estonian and the French teams. The long-term project is to develop an approach to nonlinear functional equations defined by more than one functional operators by combining the (theoretical, algorithmic) results and the implementations developed by the two teams. To demonstrate our results, we want to develop a new `Mathematica` package which generalizes the `NLControl` and OreModules packages.

Since this project is rather ambitious, the intermediate stages we want to reach first within the PARROT program are detailed below.

2.1 Theoretical issues

The aim of this work is to develop new theoretical results for the study and the control of nonlinear functional systems defined by one or more functional operators (e.g., differential, difference, shift operators).

The first point is to understand how the different results on the generic linearization of non linear systems obtained by the two teams can be extended (lifted) to the nonlinear systems (e.g., first integral of motion, conservation laws, autonomous elements, obstruction to accessibility, flatness, realization, invariants). These results will be developed by combining techniques coming from algebra, algebraic analysis [7, 13, 31, 50], differential algebra [32, 53] and difference algebra [21]. Combining the knowledge and the results of the two teams, we will certainly be able to obtain new and interesting results.

Moreover, a new approach, recently initiated by the French team in [12], for the study of certain classes of hyperbolic partial differential systems (e.g., Burgers' equation, shallow water, traffic flow, Euler equations), will be developed and applied to control theoretical problems (e.g., autonomous elements, accessibility, controllability, parametrizations, flatness) for certain classes of nonlinear functional systems (e.g., bilinear systems, quasilinear systems).

Finally, the theoretical results will be developed within a constructive approach based on symbolic computation techniques (e.g., Gröbner basis techniques, differential algebra techniques) so that they can be implemented in the existing packages and in the packages that will be developed by the project.

2.2 Implementations

One of the main objectives of the CASCAC project is to develop a new `Mathematica` package which supersedes `NLControl` and OreModules and an interface between the C library `BLAD` dedicated to differential algebra and `Mathematica`.

2.2.1 First step (2013): A `Mathematica` version of OreModules

Gröbner bases have recently been implemented by Christoph Koutschan (RICAM, Linz) in the free `Mathematica` package `HolonomicFunctions` [22] for large classes of Ore algebras. This implementation allows us to develop a `Mathematica` version of the free Maple package OreModules [14]. Using our experience in the development of the OreModules package and its generic structure, it can easily be rewritten in `Mathematica` within a few months. Combining the experience in `Mathematica` developed by the Estonian team, and particularly by Juri Belikov, Kristina Halturina, and Maris Tõnso, the experience in OreModules by Alban Quadrat and Thomas Cluzeau, and the help of Christoph Koutschan, will be the main task of the first year (2013) in the direction of implementation.

This new package will benefit from the generic implementation of Ore algebras and Gröbner bases of the `HolonomicFunctions` package. This extension is particularly suitable because the class of Ore algebras defined in the `Maple` has been stable for many years and, despite the interest of extending this class for practical applications, it seems the forthcoming

Maple releases will not have such an extension. Since the source code of Maple packages Ore_algebra and Groebner are not open, they can be extended from outside. For all these reasons, the development of a Mathematica version of OreModules is a natural project for the French team.

This new package will allow the Estonian team to extend certain commands of the NLControl package to the class of linear functional systems defined by more than one functional operators (e.g., differential time-delay systems, difference-differential systems, partial differential systems).

Finally, the new theoretical results developed during the collaboration will be implemented in this new package and a Mathematica implementation of the OreMorphisms package [19] will be started if we have enough times.

2.2.2 Second step (2014): Interface between BLAD and Mathematica

It is well known that implicit nonlinear differential systems and differential algebraic equations (DAE) are difficult systems to be handled numerically. One of the reasons comes from the fact that these systems cannot easily be rewritten in a solved form. For these reasons, such class of systems cannot be treated with the NLControl package.

Techniques of differential algebra, developed by Ritt [53] and Kolchin [32], were made constructible in [8, 9, 16, 24, 25, 26, 28] using the concept of triangular systems, coherent autoreduced sets, and characteristic sets. The corresponding algorithms were implemented by François Boulier (member of the French team) in the Maple package diffalg and in the C library BLAD [10]. Using these implementations dedicated to differential algebra techniques and to their applications in control theory and in biological systems, implicit nonlinear differential systems and differential algebraic equations can be effectively studied.

The goal of the second year (2014) of the CASCAC project will be to develop an interface between BLAD and Mathematica. This interface will allow us to extend the class of nonlinear ordinary differential systems handled by the NLControl package. In particular, the different algorithms developed by Sette Diop in [24, 25, 26] (e.g., observability, parameter identification, input-output behaviour, realization), who is a member of the French team, will be added to the NLControl package. Moreover, since differential algebra also studies polynomial partial differential systems, the interface between BLAD and Mathematica will also be used in the new package developed in the first year of the project. In particular, the new approach to quasilinear partial differential systems developed in [12] could be constructively developed using the two implementations developed in our project.

François Boulier already has a long experience in interfacing BLAD with different computer algebra systems. In particular, he developed one for Maple (the DifferentialAlgebra package). Since his contract with Maple will end in 2013, he will be able to help us in the development of a BLAD interface with Mathematica. Moreover, he is now developing another BLAD interface to the free computer algebra SAGE [57].

Finally, the interface between BLAD and Mathematica will be useful for a larger community such as the symbolic computation one which benefits from the access of differential algebra methods through the computer algebra system Mathematica.

2.3 Supervision of a Master project

The forthcoming Master project of Kristina Halturina (Estonian team) will be supervised by Ülle Kotta (Estonian team) and Alban Quadrat (French team). The subject of Kristina Halturina's Master project will be directly related to the subjects of the Estonian-French collaboration and it will particularly deal with the constructive study of a class of nonlinear differential systems called *differential flat systems* [35, 36]. This class of nonlinear systems, usually encountered in practise, can be controlled by means of motion planning and tracking approaches as explained in [35, 36]. Many attempts have been made to obtain a general test of flatness and algorithms for computing flat outputs of a flat system [1, 2, 35, 36, 40, 49, 56]. In the Master project, we shall constructively study these problems and implement the different algorithms in the `NLControl` package and in the ones developed during the collaboration.

Alban Quadrat has a long experience in the characterization of flatness and computing flat outputs in the case of multidimensional systems [27, 51, 52]. Implementations of the corresponding algorithms have been made in the OreModules [14] and Stafford packages [51].

Finally, during the Master project, the help of Hugues Mounier, a member of the French team and a world specialist on flat systems [37, 44, 45] and on industrial applications, will be precious.

3 Program of the collaboration & schedule

3.1 2013

- Study of theoretical problems on nonlinear functional systems and particularly the possibility to extend to nonlinear systems the different results based on linearization techniques (e.g., lift of autonomous elements, accessibility, controllability, parametrization, reduction) recently obtained by the two teams.
- Development of a `Mathematica` version of the OreModules package.
- Supervision of Kristina Halturina's Master project on the constructive study of differential flat nonlinear systems (characterizations of differential flat systems, computation of differential flat outputs, implementation).

3.2 2014

- Implementation of the new theoretical results on nonlinear functional systems in the new `Mathematica` package obtained during the first year.
- Development of an interface between the C library `BLAD`, dedicated to differential algebra and its applications, and the computer algebra system `Mathematica`.

4 Compositions of the teams

4.1 Estonia

1. Juri Belikov, Senior researcher,

Institute of Cybernetics, Control Systems Department, Tallinn University of Technology, Akadeemia tee 21, 12618, Tallinn, Estonia.

Tel: 00-372-620-4157
Email: jbelikov@cc ioc ee
Web: http://www.cc.ioc.ee/home_page.php?fn=Juri&ln=Belikov

2. Kristina Halturina, Master student, Technician, future PhD student,

   Institute of Cybernetics, Control Systems Department, Tallinn University of Technology, Akadeemia tee 21, 12618, Tallinn, Estonia.

3. Ülle Kotta, Leading researcher,

   Institute of Cybernetics, Control Systems Department, Tallinn University of Technology, Akadeemia tee 21, 12618, Tallinn, Estonia.

   Tel: 00-372-620-4153
   Email: kotta@cc.ioc.ee
   Web: http://www.cc.ioc.ee/home_page.php?fn=Ylle&ln=Kotta

4. Maris Tõnso, Senior researcher, **team leader**,

   Institute of Cybernetics, Control Systems Department, Tallinn University of Technology, Akadeemia tee 21, 12618, Tallinn, Estonia.

   Tel: 00-33-372-620-4188
   Email: maris@cc.ioc.ee
   Web: http://www.cc.ioc.ee/home_page.php?fn=Maris&ln=Tonso

4.2 France

1. Moulay Barkatou, Professor,

   University of Limoges, Faculté des Sciences et Techniques, XLIM ; DMI, 123 avenue Albert Thomas 87060 Limoges cedex, France.

   Tel: 00-33-(0)-5-55-45-73-83
   Email: moulay.barkatou@unilim.fr
   Web: http://www.xlim.fr/en/labo/personne?id=1572

2. François Boulier, Professor,

   Laboratoire d'Informatique Fondamentale de Lille, Université Lille I, 59655 Villeneuve d'Ascq, France.

   Tel: 00-33-(0)3-20-33-77-10
   Email: Francois.Boulier@univ-lille1.fr
   Web: http://www.lifl.fr/~boulier/pmwiki/pmwiki.php.

3. Thomas Cluzeau, Assistant Professor,

   University of Limoges, Faculté des Sciences et Techniques, XLIM ; DMI, 123 avenue Albert Thomas, 87060 Limoges cedex, France.

   Tel: 00-33-(0)5-87-50-67-74

Email: cluzeau@ensil.unilim.fr
Web: http://perso.ensil.unilim.fr/~cluzeau/

4. Sette Diop, Researcher, CNRS,

   L2S, Supélec, 3 rue Joliot Curie, 91192 Gif-sur-Yvette cedex, France.

   Tel: 00-33-(0)1-69-85-17-30
   Email: diop@lss.supelec.fr
   Web: http://www.l2s.supelec.fr/perso/diop

5. Hugues Mounier, Professor, University of Paris XI (Orsay),

   L2S, Supélec, 3 rue Joliot Curie, 91192 Gif-sur-Yvette cedex, France.

   Tel: 00-33-(0)1-69-85-17-52
   Email: hugues.mounier@lss.supelec.fr
   Web: http://webpages.lss.supelec.fr/perso/hugues.mounier/indexL2S.html.

6. Alban Quadrat, Researcher, **team leader**,

   Inria Île-de-France, DISCO Projet, L2S, Supélec, 3 rue Joliot Curie, 91192 Gif-sur-Yvette cedex, France.

   Tel: 00-33-(0)1-69-85-17-75
   Email: alban.quadrat@inria.fr
   Web: http://pages.saclay.inria.fr/alban.quadrat/.

7. Jalouli Ashraf, PhD student (starting 09/2012),

   University of Limoges, Faculté des sciences et techniques, XLIM ; DMI, 123 avenue Albert Thomas, 87060 Limoges cedex, France.

8. Suzy Maddah, PhD student (starting 09/2012)

   University of Limoges, Faculté des Sciences et Techniques, XLIM ; DMI, 123 avenue Albert Thomas, 87060 Limoges cedex, France.

5 Complementarity of the teams

5.1 French team

- Moulay Barkatou is a world specialist on linear functional systems (e.g., systems of ordinary differential equations, of difference equations), skew polynomial rings of functional operators, and symbolic computation. With Eckhard Ppluegel, he is the developer of the `Maple` package [38] dedicated to the study of linear functional systems (e.g., integration, factorization, reduction, local and global information). His experience in the direction of linear functional systems will be precious for the first part of the project.

- François Boulier is a world specialist on constructive differential algebra, symbolic computation, and their applications in biological systems. In particular, he is the main developer of the packages `diffalg` and `DifferentialAlgebra` available in the commercial release of the computer algebra system `Maple` [41]. He is also the developer of the library `BLAD` [10], written is C, dedicated to differential algebra techniques. He also has a good knowledge in control theory. His experience in the development of packages of differential algebra and in interfacing BLAD to Maple or Sage will be important for the success of the second part of the project (interfacing BLAD to Mathematica).

- Thomas Cluzeau is a world specialist in linear functional systems (e.g., systems of ordinary differential equations, of difference equations), unimodular methods, differential algebra, and symbolic computation. He also has a good knowledge in control theory (teaching, exercise classes, applications of some of his work). In collaboration with Alban Quadrat, he is the developer of the Maple package OreMorphisms [19] dedicated to the study of symmetries, factorization, reduction, and decomposition of linear functional systems. His long experience symbolic methods in Ore algebras and differential algebra will be precious for the project.

- Sette Diop is a specialist in control theory (nonlinear systems, observability, identification, (numerical) observers, decision methods) and differential algebra methods. He was the funder of the use of differential algebra in the study of nonlinear control systems. His first implementation of differential algebra in the computer algebra system Reduce yielded the study of differential algebra techniques within the symbolic computation community and particularly the work of François Boulier. His long experience in nonlinear control systems and differential algebra will be useful for the second part of the project.

- Hugues Mounier is a specialist in control systems theory (nonlinear systems, differential time-delay systems, flatness, applications). His experience in algebraic systems theory and in industrial applications of nonlinear control theory will be useful for the development of the first part of the project. Moreover, as a specialist in flat systems, he will help Ülle Kotta and Alban Quadrat in their supervisions of Kristina Halturina's Master project on this topic.

- Alban Quadrat is a world specialist on constructive algebra, symbolic computation and control theory. He is an associate editor of the Springer journal "Multidimensional Systems and Signal Processing" for which he is in charge of the section on systems theory and symbolic computation. With Daniel Robertz (University of Aachen), he is the developer of the Maple package OreModules [14] dedicated to the symbolic study of Ore algebras of functional operators (e.g., Gröbner bases), systems theory over Ore algebras (e.g., differential systems, delay systems, discrete systems), control theory (analysis and synthesis problems). In collaboration with Thomas Cluzeau, he is also a developer of the Maple package OreMorphisms [19]. His experience in both control theory and symbolic computation (Gröbner bases, differential algebra) will be central for the success of the project.

- Jalouli Ashraf and Suzy Maddah are two new PhD thesis students who will work on linear functional systems. They will benefit from the experience of the members of the Estonian team in the direction of skew polynomial rings, control theory and implementation.

5.2 Estonian team

- Juri Belikov - His theoretical studies are related to the application of the noncommutative polynomial approach to model matching problems and realization, linear parameter-varying systems. He is one of the developers of `NLControl` package [46] (model matching problems, Dieudonné determinant, Jacobson form). Moreover, he knows really well the web programming tools and is the main developer of NLControl website. Finally, he has experience in web programming environments like HTML, php, JavaScript and Java.

- Kristina Halturina is currently studing differential flatness problem and computation of the flat outputs. She is the co-developer of `NLControl package` [46].

- Ülle Kotta is a specialist in control theory and has extensive knowledge in nonlinear control systems. She has studied a large number of nonlinear control synthesis, analysis and modeling problems, including, but not limited to, accessibility, reduction, feedback linearization, system (transfer and input-output) equivalence, and transformations between different system representations (state equations, input-output equations, transfer function). She has also studied the unification of discrete- and continuous-time systems using time-scale calculus and pseudo-linear algebra.

- Maris Tõnso is a specialist in noncommutative polynomials, realization, irreducibility/accessibility, row-reduction. She masters the methods for transforming the nonlinear systems between different representations (state-space, input-output equations and transfer matrix), and for finding the minimal representation of the system. She has got programming experience in systems defined on time scales, systems given in terms of pseudo-linear algebra and in time-delay systems. She is an experienced Mathematica programmer and a leading developer of `NLControl` package [46].

6 Advantages for the teams

The success of this project highly depends on the complementarity of the two teams. None of the two teams could fufill this project on its own and the specific expertise required by the project cannot be found anywhere else.

The Estonian team will bring their expertise in the following domains:

- Constructive methods for analysis and synthesis problems of nonlinear functional systems.

- Expertise in the development of `Mathematica` packages.

- Expertise in the development of web interfaces.

- Access to well trained students in the fields of the project.

The French team will bring their expertise in the following domains:

- Expertise in algebraic methods for Ore algebras of functional operators, module theory, homological algebra, and symbolic computation.

- Expertise in Gröbner basis techniques for Ore algebras of functional operators.

- Expertise in differential algebra techniques and their applications to control theory.
- Expertise in decision methods for nonlinear control theory.
- Expertise in efficient implementations.

7 Exchange details

- **Estonia to France**
  1. 2013: Juri Belikov, Kristina Halturina, Ülle Kotta, and Maris Tõnso will visit the French team for 5 days.
  2. 2014: Juri Belikov, Kristina Halturina, Ülle Kotta, and Maris Tõnso will visit the French team for 5 days.
- **France to Estonia**
  1. 2013: Moulay Barkatou, Thomas Cluzeau, Hugues Mounier, and Alban Quadrat will visit the Estonian team for 5 days.
  2. 2014: Jalouli Ashraf, François Boulier, Sette Diop, and Alban Quadrat will visit the Estonian team for 5 days.

References

[1] D. Avanessoff J.-P. Pomet, "Flatness and Monge parameterization of two-input systems, control-affine with 4 states or general with 3 states", *ESAIM-COCV*, 13 (2007), 237-264. 5

[2] E. Aranda-Bricaire, C. H. Mooh, J.-P. Pomet, "A Linear Algebraic Framework for Dynamic Feedback Linearization ", *IEEE Trans. on Automatic Control*, 40 (1995), 127-132. 5

[3] T. Becker, V. Weispfenning, *Gröbner Bases. A Computational Approach to Commutative Algebra*, Springer, NewYork, 1993. 2

[4] J. Belikov, U. Kotta, M. Tonso, "Symbolic polynomial tools for nonlinear control systems", in *Proceedings of the Vienna Conference on Mathematical Modelling*, volume 7, Vienna University of Technology, 1-6. 2

[5] J. Belikov, U. Kotta, M. Tonso, "Minimal realization of nonlinear MIMO equations in state-space form: polynomial approach", *Proceedings of the 50th IEEE Conference on Decision and Control and European Control Conference*, Orlando, USA, 2011, 7735-7740. 2

[6] J. Belikov, U. Kotta, A. Leibak, "Transfer matrix and its Jacobson form for nonlinear systems on time scales: Mathematica implementation", *AT&P Journal PLUS*, 2 (2011), 6-12. 2

[7] A. Borel et al., *Algebraic D-modules*, Perspectives in Mathematics 2, Academic Press, 1987. 3

[8] F. Boulier, “An optimization of Seidenberg’s elimination algorithm in differential algebra”, *Mathematics and Computers in Simulation*, 42 (1996), 439-448. 4

[9] F. Boulier, D. Lazard, F. Ollivier, M. Petitot, “Computing representations for radicals of finitely generated differential ideals”, *Appl. Algebra Engrg. Comm. Comput.*, 20 (2009), 73-121. 4

[10] F. Boulier, BLAD: *Bibliothèques Lilloises d’Algèbre Différentielle*, http://www.lifl.fr/~boulier/pmwiki/pmwiki.php/Main/BLAD. 4, 7

[11] B. Buchberger, “An Algorithm for Finding the Basis Elements in the Residue Class Ring Modulo a Zero Dimensional Polynomial Ideal”, *Journal of Symbolic Computation*, 41 (2006), 475-511 (PhD Thesis, University of Innsbruck, 1965, english translation). 2

[12] A. Chakhar, C. Cluzeau, A. Quadrat, “An algebraic analysis approach to certain classes of nonlinear partial differential systems”, *Proceedings of nDS’11*, Poitiers, France, 2011. 3, 4

[13] F. Chyzak, A. Quadrat, D. Robertz, “Effective algorithms for parametrizing linear control systems over Ore algebras”, *Appl. Algebra Engrg. Comm. Comput.*, 16 (2005), 319-376. 2, 3

[14] F. Chyzak, A. Quadrat, D. Robertz, “OreModules: A symbolic package for the study of multidimensional linear systems”, in *Applications of Time-Delay Systems*, J. Chiasson and J. -J. Loiseau (Eds.), Lecture Notes in Control and Inform. 352, Springer, 2007, 233-264, OreModules project: http://wwwb.math.rwth-aachen.de/OreModules. 2, 3, 5, 8

[15] F. Chyzak, B. Salvy, “Non-commutative elimination in Ore algebras proves multivariate identities”, *J. Symbolic Comput.*, 26 (1998), 187-227. 2

[16] T. Cluzeau, E. Hubert, “Resolvent representation for regular differential ideals”, *Appl. Algebra Engrg. Comm. Comput.*, 13 (2003), 395-425. 4

[17] T. Cluzeau, A. Quadrat, “Factoring and decomposing a class of linear functional systems”, *Linear Algebra Appl.*, 428 (2008), 324-381. 2

[18] T. Cluzeau, A. Quadrat, “On algebraic simplifications of linear functional systems”, to appear in the book *Topics in Time-Delay Systems: Analysis, Algorithms and Control*, J.-J. Loiseau, W. Michiels, S.-I. Niculescu and R. Sipahi (Eds.), Lecture Notes in Control and Information Sciences (LNCIS), Springer, 2009, 167-178. 2

[19] T. Cluzeau, A. Quadrat, “OreMorphisms: A homological algebraic package for factoring and decomposing linear functional systems”, in *Topics in Time-Delay Systems: Analysis, Algorithms and Control*, J.-J. Loiseau, W. Michiels, S.-I. Niculescu and R. Sipahi (Eds.), Lecture Notes in Control and Inform. 388, Springer, 2009, 179-196, OreMorphisms project: http://pages.saclay.inria.fr/alban.quadrat/OreMorphisms.html. 2, 4, 8

[20] CoCoA System, Computations in Commutative Algebra, http://cocoa.dima.unige.it/. 1

[21] R. M. Cohn, *Difference algebra*, R.E. Krieger Pub. Co, 1979. 3

[22] C. Koutschan, HolonomicFunctions, http://www.risc.jku.at/research/combinat/software/HolonomicFunctions/. 3

[23] G. Culianez, *Formes de Hermite et de Jacobson: Implémentations et applications*, internship (INSA de Toulouse) under the supervision of A. Quadrat, INRIA Sophia Antipolis (06-07/05), Jacobson project: http://pages.saclay.inria.fr/alban.quadrat/Stages.html.

[24] S. Diop, "Finite morphisms of differential algebraic varieties and elimination theory", in *Progr. Systems Control Theory*, 8, Birkhäuser, 1991,193-200. 4

[25] S. Diop, "Elimination in control theory", *Math. Control Signals Systems*, 4 (1991), 17-32. 4

[26] S. Diop, "Differential-algebraic decision methods and some applications to system theory", *Theoret. Comput. Sci.*, 98 (1992), 137-161. 4

[27] A. Fabiańska, A. Quadrat, "Applications of the Quillen-Suslin theorem in multidimensional systems theory", in *Gröbner Bases in Control Theory and Signal Processing*, H. Park and G. Regensburger (Eds.), Radon Series on Computation and Applied Mathematics 3, de Gruyter publisher, 2007, 23-106, QuillenSuslin project: http://wwwb.math.rwth-aachen.de/QuillenSuslin/. 5

[28] E. Hubert, "Factorization free decomposition algorithms in differential algebra", *Journal of Symbolic Computations*, 29 (2000), 641-662. 4

[29] T. Kailath, *Linear Systems*, Prentice-Hall, 1980. 1, 2

[30] R. E. Kalman, P. L. Falb, M. A. Arbib, *Topics in Mathematical System Theory*, McGraw-Hill, 1969. 1, 2

[31] M. Kashiwara, *Algebraic Study of Systems of Partial Differential Equations*, Master Thesis, Tokyo Univ. 1970, Mémoires de la Sociétée Mathématiques de France 63 (1995) (English translation). 3

[32] E. R. Kolchin, *Differential Algebra and Algebraic Groups*, Academic Press, 1973. 3, 4

[33] U. Kotta, M. Tonso, "Realization of discrete-time nonlinear input-output equations: polynomial approach", *Automatica*, 48 (2012), 255-262. 2

[34] M. Fliess, "Some basic structural properties of generalized linear systems", *Systems Control Lett.*, 15 (1990), 391-396. 1, 2

[35] M. Fliess, J. Lévine, P. Martin, P. Rouchon, "Flatness and defect of nonlinear systems: introductory theory and examples", *Int. J. Control*, 61 (1995), 1327-1361. 5

[36] M. Fliess, J. Lévine, P. Martin, P. Rouchon, "A Lie-Bäcklund approach to equivalence and flatness of nonlinear systems", *IEEE Trans. Automat. Control*, 44 (1999), 922-937. 5

[37] M. Fliess, H. Mounier, "Controllability and observability of linear delay systems: an algebraic approach", *ESAIM Control Optim. Calc. Var.*, 3 (1998), 301-314. 2, 5

[38] ISOLDE, http://isolde.sourceforge.net/. 7

[39] V. Kučera, *Discrete Linear Control: The Polynomial Equation Approach*, Wiley, Chichester 1979. 1, 2

[40] J. Lévine, *Analysis and Control of Nonlinear Systems: A Flatness-Based Approach*, Springer 2009. 5

[41] Maple, http://www.maplesoft.com/products/Maple/index.aspx?L=E. 1, 2, 7

[42] Mathematica, http://www.wolfram.com/mathematica/. 1, 2

[43] J. C. McConnell, J. C. Robson, *Noncommutative Noetherian Rings*, American Mathematical Society, 2000. 1

[44] H. Mounier, P. Rouchon, J. Rudolph, "Some examples of linear systems with delays", *European Journal of Automation*, 31 (1997), 911-925. 2, 5

[45] H. Mounier, J. Rudolph, M. Fliess, P. Rouchon, "Tracking control of a vibrating string with an interior mass viewed as delay system", *ESAIM Control Optim. Calc. Var.*, 3 (1998), 315-321. 2, 5

[46] NLControl, http://www.cc.ioc.ee/software.php. 2, 9

[47] O. Ore, "Theory of non-commutative polynomials", *Ann. of Math.*, 34 (1933), 480-508. 1

[48] J. W. Polderman, J. C. Willems, *Introduction to Mathematical Systems Theory. A Behavioral Approach*, Texts in Mathematics 26, Springer, 1998. 1, 2

[49] J.-B. Pomet, "On dynamic feedback linearization of four-dimensional affine control systems with two inputs", *ESAIM-COCV*, 2 (1997), 151–230. 5

[50] J.-F. Pommaret, A. Quadrat, "Algebraic analysis of linear multidimensional control systems", *IMA J. Math. Control Inform.*, 16 (1999), 275-297. 2, 3

[51] A. Quadrat, D. Robertz, "Computation of bases of free modules over the Weyl algebras", *J. Symbolic Comput.*, 42 (2007), 1113-1141, Stafford project (http://wwwb.math.rwth-aachen.de/OreModules). 2, 5

[52] A. Quadrat, D. Robertz, "Controllability and differential flatness of linear analytic ordinary differential systems", *Proceedings of MTNS 2010*, Budapest (Hungary) (05-07/07/10), in *Algebraic Systems Theory, Behaviors, and Codes*, E. Zerz (Eds.), Shaker, 2010, 23-30. 5

[53] J. F. Ritt, *Differential Algebra*, RI: Amer. Math. Soc., 1950. 3, 4

[54] H. H. Rosenbrock, *State Space and Multivariable Theory*, Wiley, 1970. 1, 2

[55] J. J. Rotman, *An Introduction to Homological Algebra*, 2nd edition, Springer, 2009. 2

[56] P. Rouchon, "Necessary condition and genericity of dynamic feedback linearization", *J. Math. Systems Estim. Control*, 4 (1994), 1-14. 5

[57] `SAGE`: Mathematics software, `http://www.sagemath.org/`. 4

[58] M. Tonso, H. Rennik, U. Kotta, "WebMathematica-based tools for discrete-time nonlinear control systems", *Proceedings of the Estonian Academy of Sciences*, 58 (2009), 224-240. 2

[59] E. Zerz, *Topics in Multidimensional Linear Systems Theory*, Lecture Notes in Control and Information Sciences 256, Springer, 2000. 2