```
> restart:
> with(LinearAlgebra):
> with(OreModules):
> march('open', "/Users/aquadrat/Documents/Travail/maple/Personnel/RankFactorizationProblem.mla"):
```

libname := "/Users/aquadrat/Documents/Travail/maple/Personnel", "/Library/Frameworks/Maple.framework/Versions/2023/lib", "/Users/aquadrat/Documents/Travail/maple/Personnel/RankFactorizationProblem.mla" **(1)**

```
> with(RankFactorizationProblem);
```

[*AntiDiagonal*, *CentroHermitian*, *Factorization*, *FiniteFreeResolution*, *FittingIdeal*, *IsCentroHermitian*, *IsInvertible*, *IsNilpotent*, *IsSolution*, *LeeMatrix*, *LeftLift*, *Lift*, *LocalSyzygyModule*, *RankFactorization*, *ReducedSyzygies*, *RightLift*, *Saturation*, *Simplification*, *Solutions*, *Syzygies*] **(2)**

Antidiagonal matrices

```
> AntiDiagonal(1);
```

$$\begin{bmatrix} 1 \end{bmatrix} \tag{3}$$

```
> AntiDiagonal(2);
```

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{4}$$

```
> AntiDiagonal(3);
```

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{5}$$

```
> AntiDiagonal(4);
```

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

Lee's matrices

```
> LeeMatrix(2);
```

$$\begin{bmatrix} 1 & I \\ 1 & -I \end{bmatrix} \qquad (7)$$

```
> MatrixInverse(%);
```

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{I}{2} & \frac{I}{2} \end{bmatrix} \qquad (8)$$

```
> LeeMatrix(3);
```

$$\begin{bmatrix} 1 & 0 & I \\ 0 & 1 & 0 \\ 1 & 0 & -I \end{bmatrix} \qquad (9)$$

```
> MatrixInverse(%);
```

$$\begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ -\frac{I}{2} & 0 & \frac{I}{2} \end{bmatrix} \qquad (10)$$

```
> L := LeeMatrix(4);
```

$$L := \begin{bmatrix} 1 & 0 & I & 0 \\ 0 & 1 & 0 & I \\ 0 & 1 & 0 & -I \\ 1 & 0 & -I & 0 \end{bmatrix} \qquad (11)$$

```
> MatrixInverse(L);
```

$$\begin{bmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{I}{2} & 0 & 0 & \frac{I}{2} \\ 0 & -\frac{I}{2} & \frac{I}{2} & 0 \end{bmatrix} \qquad (12)$$

Test whether or not L is centrohermitian

```
> IsCentroHermitian(L);
```

$$false \quad (13)$$

Compute the centrohermitian matrix associated with L

```
> H := CentroHermitian(L);
```

$$H := \begin{bmatrix} \frac{1}{2} & \frac{I}{2} & \frac{I}{2} & \frac{1}{2} \\ \frac{I}{2} & \frac{1}{2} & \frac{1}{2} & \frac{I}{2} \\ -\frac{I}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{I}{2} \\ \frac{1}{2} & -\frac{I}{2} & -\frac{I}{2} & \frac{1}{2} \end{bmatrix} \quad (14)$$

```
> IsCentroHermitian(H);
```

$$true \quad (15)$$

We can check that L is not unitary

```
> simplify(Transpose(conjugate(L)).L);
```

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \quad (16)$$

Unitary Lee's matrices

```
> M := LeeMatrix(4,"unitary");
```

(17)

$$M := \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{I}{2}\sqrt{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 & \frac{I}{2}\sqrt{2} \\ 0 & \frac{\sqrt{2}}{2} & 0 & -\frac{I}{2}\sqrt{2} \\ \frac{\sqrt{2}}{2} & 0 & -\frac{I}{2}\sqrt{2} & 0 \end{bmatrix} \quad (17)$$

```
> simplify(Transpose(conjugate(M)).M);
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (18)$$

Unitary Lee's matrices with a parameter

```
> u := 'u';
```

$$u := u \quad (19)$$

```
> R := LeeMatrix(4,"unitary_symbolic",u);
```

$$R := \left[\begin{bmatrix} \frac{1}{u} & 0 & \frac{I}{u} & 0 \\ 0 & \frac{1}{u} & 0 & \frac{I}{u} \\ 0 & \frac{1}{u} & 0 & \frac{-I}{u} \\ \frac{1}{u} & 0 & \frac{-I}{u} & 0 \end{bmatrix}, u, u^2 - 2 \right] \quad (20)$$

Lee's bijective transformation sending a centrohermitian M to a real matrix M_rho (square case)

```
> M := Matrix([[9+18*I,-225,9+198*I],[0,0,0],[9-198*I,-225,9-18*I]]);
```

$$M := \begin{bmatrix} 9 + 18\,I & -225 & 9 + 198\,I \\ 0 & 0 & 0 \\ 9 - 198\,I & -225 & 9 - 18\,I \end{bmatrix} \quad (21)$$

```
> IsCentroHermitian(M);
```

$$true \quad (22)$$

```
> U := LeeMatrix(3);
```

$$U := \begin{bmatrix} 1 & 0 & I \\ 0 & 1 & 0 \\ 1 & 0 & -I \end{bmatrix} \quad (23)$$

```
> U_inv := MatrixInverse(U);
```

$$U\_inv := \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ -\frac{I}{2} & 0 & \frac{I}{2} \end{bmatrix} \quad (24)$$

```
> M_rho := U_inv.M.U;
```

$$M\_rho := \begin{bmatrix} 18 & -225 & 180 \\ 0 & 0 & 0 \\ 216 & 0 & 0 \end{bmatrix} \quad (25)$$

```
> U.M_rho.U_inv;
```

$$\begin{bmatrix} 9+18\,I & -225 & 9+198\,I \\ 0 & 0 & 0 \\ 9-198\,I & -225 & 9-18\,I \end{bmatrix} \quad (26)$$

```
> simplify(%-M);
```

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (27)$$

Lee's bijective transformation sending a centrohermitian M to a real matrix M_rho (rectangular case)

```
> M := Matrix(2, 5, [[-29, 0, -26, -6*I, -56*I], [56*I, 6*I, -26, 0,
  -29]]);
```

$$M := \begin{bmatrix} -29 & 0 & -26 & -6\,I & -56\,I \\ 56\,I & 6\,I & -26 & 0 & -29 \end{bmatrix} \quad (28)$$

```
> IsCentroHermitian(M);
```

$$true \quad (29)$$

```
> U := LeeMatrix(2);
```

$$U := \begin{bmatrix} 1 & I \\ 1 & -I \end{bmatrix} \quad (30)$$

```
> U_inv := MatrixInverse(U);
```

$$U\_inv := \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{I}{2} & \frac{I}{2} \end{bmatrix} \quad (31)$$

```
> V := LeeMatrix(5);
```

$$V := \begin{bmatrix} 1 & 0 & 0 & I & 0 \\ 0 & 1 & 0 & 0 & I \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -I \\ 1 & 0 & 0 & -I & 0 \end{bmatrix} \quad (32)$$

```
> V_inv := MatrixInverse(V);
```

$$V\_inv := \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -\frac{I}{2} & 0 & 0 & 0 & \frac{I}{2} \\ 0 & -\frac{I}{2} & 0 & \frac{I}{2} & 0 \end{bmatrix} \quad (33)$$

```
> M_rho := U_inv.M.V;
```

$$M\_rho := \begin{bmatrix} -29 & 0 & -26 & -56 & -6 \\ -56 & -6 & 0 & -29 & 0 \end{bmatrix} \quad (34)$$

```
> U.M_rho.V_inv;
```

$$\begin{bmatrix} -29 & 0 & -26 & -6\,I & -56\,I \\ 56\,I & 6\,I & -26 & 0 & -29 \end{bmatrix} \quad (35)$$

```
> simplify(%-M);
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (36)$$