Let us consider the first Weyl algebra $A = A_1(\mathbb{Q})$, where $\mathbb{Q}$ is the field of rational numbers,

```plaintext
> A := DefineOreAlgebra(diff=[d,t], polynom=[t]);
```

and the left $A$-module $M$ finitely presented by the matrix $R$ defined by:

```plaintext
> R := evalm([[0,d,0,-1],[d,0,-t,0]]);
```

The rank of the finitely presented left $A$-module $M$ is:

```plaintext
> OreRank(R,A);
```

2

Since $R$ admits a right inverse $S$ defined by

```plaintext
> S := RightInverse(R,A);
```

$$
S := \begin{bmatrix}
0 & t \\
0 & 0 \\
0 & d \\
-1 & 0
\end{bmatrix}
$$

$M$ is a stably free left $A$-module of rank 2, i.e., a free left $A$-module of rank 2. Using the fact that the direct sum of $A^1 \times 2$ and $M$ is isomorphic to $A^1 \times 4$, and using the Cancellation Theorem, let us compute a basis of $M$. Let us first compute

```plaintext
> X := stackmatrix(R,1-Mult(S,R,A));
```

which defines the left $A$-isomorphism $g$ from the direct sum of $A^1 \times 2$ and $M$ onto $A^1 \times 4$. Moreover, the direct sum of $A^1 \times 2$ and $M$ is isomorphic to the left $A$-module $L$ finitely presented by the matrix $P = (0 \ R)$ defined by

```plaintext
> P := augment(evalm([[0,0],[0,0]]),R);
```

Similarly, a finite presentation of $A^1 \times 3$ is given by the matrix $R'$ defined by

```plaintext
> Rp := evalm([[0$3]]);
```
\[ Rp := \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \]

and \( A^{1 \times 4} \) is isomorphic to the left \( A \)-module \( L' \) finitely presented by the matrix \( P' = (0 \ R') \) defined by

\[
Pp := \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}
\]

Let us check again that the left \( A \)-homomorphism \( f \) from \( L \) to \( L' \) induced by \( X \) is a left \( A \)-isomorphism:

\[
\text{TestIso}(P, Pp, X, A);
\]

\[ true \]

We obtain a left \( A \)-isomorphism \( h_1 \) from \( M_1 \) onto \( A^{1 \times 3} \) induced by \( Q_1 \), where \( M_1 \) is the left \( A \)-module finitely presented by \( P_1 \), which is isomorphic to the direct sum of \( A \) and \( M \), and the matrix \( P_1 = (0 \ R) \) is defined by

\[
P_1 := \begin{bmatrix} 0 & 0 & d & 0 & -d & 0 & -1 \\ 0 & d & 0 & -t & 0 \end{bmatrix}
\]

and the matrix \( Q_1 \) is defined by

\[
Q_1 := \begin{bmatrix}
d^2 & -t & -d \\
-(d t - 1) d & t^2 & d t - 1 \\
1 & 0 & 0 \\
-d^3 & 2 + d t & d^2 \\
d & 0 & 0
\end{bmatrix}
\]

Let us check again that \( h_1 \) is a left \( A \)-isomorphism:

\[
\text{TestIso}(P_1, Rp, Q_1, A);
\]

\[ true \]

Let \( R'' = (0 \ 0) \), namely,

\[
Rpp := \text{evalm([[0$2]])};
\]

\[ Rpp := \begin{bmatrix} 0 & 0 \end{bmatrix} \]

and \( L'' \) be the left \( A \)-module finitely presented by \( R'' \), which is isomorphic to \( A^{1 \times 2} \). We obtain a left \( A \)-isomorphism \( h_2 \) from \( M \) onto \( A^{1 \times 2} \) induced by \( Q_2 \), where \( Q_2 \) is the matrix defined by:

\[
Q_2 := \begin{bmatrix}
t^2 + d^2 t^2 + d t - 1 & d t - 1 + (d t - 1) d^2 \\
-t & -d \\
2 + d t + d^3 t + 3 d^2 & d^2 + d^4 \\
-1 - d t & -d^2
\end{bmatrix}
\]

Thus, \( Q_2 \) is an injective parametrization of \( M \), namely, \( \ker_A(Q_2) = A^{1 \times 2} \ R \),

\[ 2 \]
or equivalently, \( M \) is isomorphic to \( A^{1 \times 4} \) \( Q_2 \), and \( Q_2 \) admits a left inverse defined by

\[
T_2 := \begin{bmatrix}
0 & 0 & 1 & 1 + d^2 \\
-1 & -t & 0 & -dt + 1
\end{bmatrix}
\]