Let us consider the first Weyl algebra $A = A_1(\mathbb{Q})$, where $\mathbb{Q}$ is the field of rational numbers,

\begin{verbatim}
> A := DefineOreAlgebra(diff=[d,t], polynom=[t]):
\end{verbatim}

and the left $A$-module $M$ finitely presented by the matrix defined by:

\begin{verbatim}
> R := evalm([[d,t,0]]):
R := [ d  t  0 ]
\end{verbatim}

The rank of the left $A$-module $M$ is:

\begin{verbatim}
> OreRank(R,A);
2
\end{verbatim}

Since $R$ admits a right inverse $S$ defined by

\begin{verbatim}
> S := RightInverse(R,A);
S := [ t 
      [ -d 
        0 ]
\end{verbatim}

$M$ is a stably free left $A$-module of rank 2, i.e., a free left $A$-module of rank 2. Using the fact that the direct sum of $M$ and $A$ is isomorphic to $A^{1 \times 3}$, and using the Cancellation Theorem, let us compute a basis of $M$. Let us first compute

\begin{verbatim}
> X := stackmatrix(R,1-Mult(S,R,A));
X := [ d  t  0
      [ -dt+1 -t^2  0
        d^2 2+dt  0
      0  0  1 ]
\end{verbatim}

which defines a left $A$-isomorphism from the direct sum of $A$ and $M$ onto $A^{1 \times 3}$. Moreover, the direct sum of $A$ and $M$ is isomorphic to the left $A$-module $L$ finitely presented by

\begin{verbatim}
> P := augment(evalm([[0]]),R);
P := [ 0  d  t  0 ]
\end{verbatim}

Similarly, a finite presentation of $A^{1 \times 2}$ is given by the matrix defined by

\begin{verbatim}
> Rp := evalm([[0$2]]);
Rp := [ 0  0 ]
\end{verbatim}

and $A^{1 \times 3}$ is isomorphic to the left $A$-module $L'$ which is finitely presented by the matrix defined by

\begin{verbatim}
> Pp := augment(evalm([[0]]),Rp);
\end{verbatim}
Let us check again that the left $\mathcal{A}$-homomorphism from $L$ to $L'$ induced by $X$ is a left $\mathcal{A}$-isomorphism:

\[
\texttt{TestIso(P,Pp,X,A);}
\]

\[
\texttt{true}
\]

We can now apply the algorithm corresponding to the Cancellation Theorem to these matrices:

\[
\texttt{Q := map(collect,Cancellation(Rp,X,A,"splithom"),[d,t]);}
\]

to obtain a left $\mathcal{A}$-isomorphism $h$ from $M$ to $\mathcal{A}^{\text{1 x 2}}$ induced by $Q$

\[
\texttt{rowdim(Q); coldim(Q);}
\]

\[
3 \quad 2
\]

whose first column is defined by

\[
\texttt{submatrix(Q,1..3,1..1);}
\]

\[
\begin{bmatrix}
(-t^4 - t^3) d^2 + (-3 t^2 - 4 t^3 - t^4) d - 3 t^2 - 2 t^3 \\
(t^2 + t^3) d^3 + (t^3 + 6 t + 8 t^2) d^2 + (15 t + 6 t^2 + 6) d + 6 + 6 t \\
-2 d^2 t^3 + (-3 t^2) d - t - 2 t^2
\end{bmatrix}
\]

and whose second columns is defined by

\[
\texttt{submatrix(Q,1..3,2..2);}
\]

\[
\begin{bmatrix}
(-t^4 - t^2 - 2 t^3) d^3 + (-t - t^4 - 6 t^2 - 6 t^3) d^2 + (-6 t^2 - 2 t + 1 - 3 t^3) d - t^2 + 2 \\
2 + (t + 2 t^2 + t^3) d^4 + (3 + 10 t^2 + 12 t + t^3) d^3 + (14 + 24 t + 7 t^2) d^2 + (10 t + 12) d \\
-2 d^3 t^3 + (-6 t + 5 t^2 - t^3) d^2 + (3 t - 3 t^2) d + 1 - t
\end{bmatrix}
\]

Let us check again that $h$ is a left $\mathcal{A}$-isomorphism:

\[
\texttt{TestIso(R,Rp,Q,A);}
\]

\[
\texttt{true}
\]

Thus, the matrix $Q$ defines an injective parametrization of $M$, i.e., we have $\ker_{\mathcal{A}} Q = \mathcal{A} R$

\[
\texttt{SyzygyModule(Q,A);}
\]

\[
\begin{bmatrix}
d & t & 0
\end{bmatrix}
\]

and $Q$ admits a left inverse $T$

\[
\texttt{T := map(collect,LeftInverse(Q,A),[d,t]);}
\]

\[
\texttt{rowdim(T); coldim(T);}
\]

\[
2 \quad 3
\]

whose first two columns are

\[
\texttt{submatrix(T,1..2,1..2);}
\]
\[-2 (t^2 + t^3)d^2 + (5t + 7t^2 + t^3)d + 4 + 10t + 5t^2\]
\[t\]
\[-2t^2 - t^3 - t^3d\]

and whose last column is defined by

\[\text{submatrix}(T,1..2,3..3);\]
\[
\begin{bmatrix}
(t + 2t^2 + t^3)d^3 + (3 + 11t + 9t^2 + t^3)d^2 + (11 + 19t + 6t^2)d + 7 + 6t \\
(-t^3 - t^2)d + (-t^3 - t^2)d + 1 - 2t - 2t^2
\end{bmatrix}
\]

Hence, we get the exact sequence

\[0 \rightarrow A \xrightarrow{R} A^{1 \times 3} \xrightarrow{Q} A^{1 \times 2} \rightarrow 0,\]

which shows that M, defined by \(\text{coker}_A(.R)\), is isomorphic to \(\text{im}_A(Q) = A^{1 \times 2}\). In particular, the residue classes of the rows of the left inverse T of Q define a basis of M.