Tank model 1

Let us introduce the following Ore algebra A of differential time-delay operators

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A = OreAlgebra[Der[t], S[t]]
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$\mathbb{K}(t)[D_t; 1, D_t][S_t; S_t, 0]$

and the matrix R of ordinary differential time-delay operators which defines a tank model described in Dubois, F., Petit, N. and Rouchon, P.,
Motion planning and nonlinear simulations for a tank containing a fluid, Proceedings of the 5th European Control Conference, Karlsruhe (Germany), 1999.

```
MatrixForm[
 R = ToOrePolynomial[{{S[t]^2, 1, -2 Der[t] S[t]}, {1, S[t]^2, -2 Der[t] S[t]}}, A]]
```

$$\begin{pmatrix} S_t^2 & 1 & -2 D_t S_t \\ 1 & S_t^2 & -2 D_t S_t \end{pmatrix}$$

and the left A-module, finitely presented by R, defined by M = $A^{1\times 3} / A^{1\times 2} R$.
Let us check whether or not M can be decomposed. We first compute the endomorphisms of M defined a constant matrix P:

```
MatrixForm[P = Morphisms[R, R, {0, 0}, p, A]]
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$$\begin{pmatrix} p[1][1] & p[2][1] & 0 \\ p[2][1] & p[1][1] & 0 \\ 0 & 0 & p[1][1] + p[2][1] \end{pmatrix}$$

From this P, let us try to compute idempotent endomorphisms of M:

```
MatrixForm /@ (PAll = IdempotentMorphisms[P, R, p, A])
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$$\left\{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right\}$$

Considering the nontrivial idempotent endomorphism PAll[[2]], we obtain the following decomposition:

```
MatrixForm[Decomposition[R, PAll[[2]], A]]
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$$\begin{pmatrix} -4 D_t S_t & S_t^2 + 1 & 0 \\ 0 & 0 & S_t^2 - 1 \end{pmatrix}$$