Einstein equations (linearized)

Example from Pommaret, J.-F. Dualite differentielle et applications, C.R. Acad. Sci. Paris, Serie I 320 (1995), pp.1225-1230.

Let us introduce the following Ore algebra A of partial differential operators

```
A = OreAlgebra[Der[t], Der[x1], Der[x2], Der[x3]]
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$\mathbb{K}(t, x_1, x_2, x_3)[D_t; 1, D_t][D_{x_1}; 1, D_{x_1}][D_{x_2}; 1, D_{x_2}][D_{x_3}; 1, D_{x_3}]$

and the matrix R of partial differential operators, which defines system

```
R = {{-Der[t]^2 + Der[x2]^2 + Der[x3]^2, Der[x1]^2, Der[x1]^2, -Der[x1]^2,
     -2 Der[x1] Der[x2], 0, 0, -2 Der[x1] Der[x3], 0, 2 Der[t] Der[x1]},
    {Der[x2]^2, -Der[t]^2 + Der[x1]^2 + Der[x3]^2, Der[x2]^2, -Der[x2]^2,
     -2 Der[x1] Der[x2], -2 Der[x2] Der[x3], 0, 0, 2 Der[t] Der[x2], 0},
    {Der[x3]^2, Der[x3]^2, -Der[t]^2 + Der[x1]^2 + Der[x2]^2, -Der[x3]^2, 0,
     -2 Der[x2] Der[x3], 2 Der[t] Der[x3], -2 Der[x1] Der[x3], 0, 0},
    {Der[t]^2, Der[t]^2, Der[t]^2, Der[x1]^2 + Der[x2]^2 + Der[x3]^2, 0, 0,
     -2 Der[t] Der[x3], 0, -2 Der[t] Der[x2], -2 Der[t] Der[x1]},
    {0, 0, Der[x1] Der[x2], -Der[x1] Der[x2], -Der[t]^2 + Der[x3]^2,
     -Der[x1] Der[x3], 0, -Der[x2] Der[x3], Der[t] Der[x1], Der[t] Der[x2]},
    {Der[x2] Der[x3], 0, 0, -Der[x2] Der[x3], -Der[x1] Der[x3],
     -Der[t]^2 + Der[x1]^2, Der[t] Der[x2], -Der[x1] Der[x2], Der[t] Der[x3], 0},
    {Der[t] Der[x3], Der[t] Der[x3], 0, 0, 0, -Der[t] Der[x2],
     Der[x1]^2 + Der[x2]^2, -Der[t] Der[x1], -Der[x2] Der[x3], -Der[x1] Der[x3]},
    {0, Der[x1] Der[x3], 0, -Der[x1] Der[x3], -Der[x2] Der[x3],
     -Der[x1] Der[x2], Der[t] Der[x1], -Der[t]^2 + Der[x2]^2, 0, Der[t] Der[x3]},
    {Der[t] Der[x2], 0, Der[t] Der[x2], 0, -Der[t] Der[x1], -Der[t] Der[x3],
     -Der[x2] Der[x3], 0, Der[x1]^2 + Der[x3]^2, -Der[x1] Der[x2]},
    {0, Der[t] Der[x1], Der[t] Der[x1], 0, -Der[t] Der[x2], 0, -Der[x1] Der[x3],
     -Der[t] Der[x3], -Der[x1] Der[x2], Der[x2]^2 + Der[x3]^2}};
```

```
R = ToOrePolynomial[R, A]
```

$$\{\{-D_t^2 + D_{x_2}^2 + D_{x_3}^2,\ D_{x_1}^2,\ D_{x_1}^2,\ -D_{x_1}^2,\ -2 D_{x_1} D_{x_2},\ 0,\ 0,\ -2 D_{x_1} D_{x_3},\ 0,\ 2 D_t D_{x_1}\},$$
$$\{D_{x_2}^2,\ -D_t^2 + D_{x_1}^2 + D_{x_3}^2,\ D_{x_2}^2,\ -D_{x_2}^2,\ -2 D_{x_1} D_{x_2},\ -2 D_{x_2} D_{x_3},\ 0,\ 0,\ 2 D_t D_{x_2},\ 0\},$$
$$\{D_{x_3}^2,\ D_{x_3}^2,\ -D_t^2 + D_{x_1}^2 + D_{x_2}^2,\ -D_{x_3}^2,\ 0,\ -2 D_{x_2} D_{x_3},\ 2 D_t D_{x_3},\ -2 D_{x_1} D_{x_3},\ 0,\ 0\},$$
$$\{D_t^2,\ D_t^2,\ D_t^2,\ D_{x_1}^2 + D_{x_2}^2 + D_{x_3}^2,\ 0,\ 0,\ -2 D_t D_{x_3},\ 0,\ -2 D_t D_{x_2},\ -2 D_t D_{x_1}\},$$
$$\{0,\ 0,\ D_{x_1} D_{x_2},\ -D_{x_1} D_{x_2},\ -D_t^2 + D_{x_3}^2,\ -D_{x_1} D_{x_3},\ 0,\ -D_{x_2} D_{x_3},\ D_t D_{x_1},\ D_t D_{x_2}\},$$
$$\{D_{x_2} D_{x_3},\ 0,\ 0,\ -D_{x_2} D_{x_3},\ -D_{x_1} D_{x_3},\ -D_t^2 + D_{x_1}^2,\ D_t D_{x_2},\ -D_{x_1} D_{x_2},\ D_t D_{x_3},\ 0\},$$
$$\{D_t D_{x_3},\ D_t D_{x_3},\ 0,\ 0,\ 0,\ -D_t D_{x_2},\ D_{x_1}^2 + D_{x_2}^2,\ -D_t D_{x_1},\ -D_{x_2} D_{x_3},\ -D_{x_1} D_{x_3}\},$$
$$\{0,\ D_{x_1} D_{x_3},\ 0,\ -D_{x_1} D_{x_3},\ -D_{x_2} D_{x_3},\ -D_{x_1} D_{x_2},\ D_t D_{x_1},\ -D_t^2 + D_{x_2}^2,\ 0,\ D_t D_{x_3}\},$$
$$\{D_t D_{x_2},\ 0,\ D_t D_{x_2},\ 0,\ -D_t D_{x_1},\ -D_t D_{x_3},\ -D_{x_2} D_{x_3},\ 0,\ D_{x_1}^2 + D_{x_3}^2,\ -D_{x_1} D_{x_2}\},$$
$$\{0,\ D_t D_{x_1},\ D_t D_{x_1},\ 0,\ -D_t D_{x_2},\ 0,\ -D_{x_1} D_{x_3},\ -D_t D_{x_3},\ -D_{x_1} D_{x_2},\ D_{x_2}^2 + D_{x_3}^2\}\}$$

and the left A-module, finitely presented by R, defined by $M = A^{1\times 10} / A^{1\times 10} R$.

Let us compute the adjoint of R:

```
Radj = Involution[R, A]
```

$$\{\{-D_t^2 + D_{x_2}^2 + D_{x_3}^2,\ D_{x_2}^2,\ D_{x_3}^2,\ D_t^2,\ 0,\ D_{x_2} D_{x_3},\ D_t D_{x_3},\ 0,\ D_t D_{x_2},\ 0\},$$
$$\{D_{x_1}^2,\ -D_t^2 + D_{x_1}^2 + D_{x_3}^2,\ D_{x_3}^2,\ D_t^2,\ 0,\ 0,\ D_t D_{x_3},\ D_{x_1} D_{x_3},\ 0,\ D_t D_{x_1}\},$$
$$\{D_{x_1}^2,\ D_{x_2}^2,\ -D_t^2 + D_{x_1}^2 + D_{x_2}^2,\ D_t^2,\ D_{x_1} D_{x_2},\ 0,\ 0,\ 0,\ D_t D_{x_2},\ D_t D_{x_1}\},$$
$$\{-D_{x_1}^2,\ -D_{x_2}^2,\ -D_{x_3}^2,\ D_{x_1}^2 + D_{x_2}^2 + D_{x_3}^2,\ -D_{x_1} D_{x_2},\ -D_{x_2} D_{x_3},\ 0,\ -D_{x_1} D_{x_3},\ 0,\ 0\},$$
$$\{-2\, D_{x_1} D_{x_2},\ -2\, D_{x_1} D_{x_2},\ 0,\ 0,\ -D_t^2 + D_{x_3}^2,\ -D_{x_1} D_{x_3},\ 0,\ -D_{x_2} D_{x_3},\ -D_t D_{x_1},\ -D_t D_{x_2}\},$$
$$\{0,\ -2\, D_{x_2} D_{x_3},\ -2\, D_{x_2} D_{x_3},\ 0,\ -D_{x_1} D_{x_3},\ -D_t^2 + D_{x_1}^2,\ -D_t D_{x_2},\ -D_{x_1} D_{x_2},\ -D_t D_{x_3},\ 0\},$$
$$\{0,\ 0,\ 2\, D_t D_{x_3},\ -2\, D_t D_{x_3},\ 0,\ D_t D_{x_2},\ D_{x_1}^2 + D_{x_2}^2,\ D_t D_{x_1},\ -D_{x_2} D_{x_3},\ -D_{x_1} D_{x_3}\},$$
$$\{-2\, D_{x_1} D_{x_3},\ 0,\ -2\, D_{x_1} D_{x_3},\ 0,\ -D_{x_2} D_{x_3},\ -D_{x_1} D_{x_2},\ -D_t D_{x_1},\ -D_t^2 + D_{x_2}^2,\ 0,\ -D_t D_{x_3}\},$$
$$\{0,\ 2\, D_t D_{x_2},\ 0,\ -2\, D_t D_{x_2},\ D_t D_{x_1},\ D_t D_{x_3},\ -D_{x_2} D_{x_3},\ 0,\ D_{x_1}^2 + D_{x_3}^2,\ -D_{x_1} D_{x_2}\},$$
$$\{2\, D_t D_{x_1},\ 0,\ 0,\ -2\, D_t D_{x_1},\ D_t D_{x_2},\ 0,\ -D_{x_1} D_{x_3},\ D_t D_{x_3},\ -D_{x_1} D_{x_2},\ D_{x_2}^2 + D_{x_3}^2\}\}$$

Let us check whether or not M is torsion-free:

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Timing[MatrixForm /@ ({Ann, Rp, Q} = Exti[Radj, A, 1])]
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We obtain 20 autonomous elements defined by Rp.

```
aut = AutonomousElements[R, Table[Ωi[t, x1, x2, x3], {i, 10}], T, A]
```

$$\{\{T[1][t, x_1, x_2, x_3] \to -\Omega_{10}{}^{(0,0,1,1)}[t, x_1, x_2, x_3] + \Omega_9{}^{(0,1,0,1)}[t, x_1, x_2, x_3] + \Omega_8{}^{(1,0,1,0)}[t, x_1, x_2, x_3] - \Omega_6{}^{(1,1,0,0)}[t, x_1, x_2, x_3],$$
$$T[2][t, x_1, x_2, x_3] \to -\Omega_9{}^{(0,1,0,1)}[t, x_1, x_2, x_3] + \Omega_7{}^{(0,1,1,0)}[t, x_1, x_2, x_3] + \Omega_5{}^{(1,0,0,1)}[t, x_1, x_2, x_3] - \Omega_8{}^{(1,0,1,0)}[t, x_1, x_2, x_3],$$
$$T[3][t, x_1, x_2, x_3] \to \Omega_4{}^{(0,0,1,1)}[t, x_1, x_2, x_3] - \Omega_9{}^{(1,0,0,1)}[t, x_1, x_2, x_3] - \Omega_7{}^{(1,0,1,0)}[t, x_1, x_2, x_3] + \Omega_6{}^{(2,0,0,0)}[t, x_1, x_2, x_3],$$
$$T[4][t, x_1, x_2, x_3] \to -\Omega_4{}^{(0,1,0,1)}[t, x_1, x_2, x_3] + \Omega_{10}{}^{(1,0,0,1)}[t, x_1, x_2, x_3] + \Omega_7{}^{(1,1,0,0)}[t, x_1, x_2, x_3] - \Omega_8{}^{(2,0,0,0)}[t, x_1, x_2, x_3],$$
$$T[5][t, x_1, x_2, x_3] \to \Omega_4{}^{(0,1,1,0)}[t, x_1, x_2, x_3] - \Omega_{10}{}^{(1,0,1,0)}[t, x_1, x_2, x_3] - \Omega_9{}^{(1,1,0,0)}[t, x_1, x_2, x_3] + \Omega_5{}^{(2,0,0,0)}[t, x_1, x_2, x_3],$$
$$T[6][t, x_1, x_2, x_3] \to \Omega_5{}^{(0,0,0,2)}[t, x_1, x_2, x_3] - \Omega_8{}^{(0,0,1,1)}[t, x_1, x_2, x_3] - \Omega_6{}^{(0,1,0,1)}[t, x_1, x_2, x_3] + \Omega_3{}^{(0,1,1,0)}[t, x_1, x_2, x_3],$$
$$T[7][t, x_1, x_2, x_3] \to \Omega_9{}^{(0,0,0,2)}[t, x_1, x_2, x_3] - \Omega_7{}^{(0,0,1,1)}[t, x_1, x_2, x_3] - \Omega_6{}^{(1,0,0,1)}[t, x_1, x_2, x_3] + \Omega_3{}^{(1,0,1,0)}[t, x_1, x_2, x_3],$$
$$T[8][t, x_1, x_2, x_3] \to -\Omega_{10}{}^{(0,0,0,2)}[t, x_1, x_2, x_3] + \Omega_7{}^{(0,1,0,1)}[t, x_1, x_2, x_3] + \Omega_8{}^{(1,0,0,1)}[t, x_1, x_2, x_3] - \Omega_3{}^{(1,1,0,0)}[t, x_1, x_2, x_3],$$
$$T[9][t, x_1, x_2, x_3] \to \Omega_4{}^{(0,0,0,2)}[t, x_1, x_2, x_3] - 2\,\Omega_7{}^{(1,0,0,1)}[t, x_1, x_2, x_3] + \Omega_3{}^{(2,0,0,0)}[t, x_1, x_2, x_3],\ T[10][t, x_1, x_2, x_3] \to -\Omega_2{}^{(0,0,0,2)}[t, x_1, x_2, x_3] + 2\,\Omega_6{}^{(0,0,1,1)}[t, x_1, x_2, x_3] - \Omega_3{}^{(0,0,2,0)}[t, x_1, x_2, x_3],$$
$$T[11][t, x_1, x_2, x_3] \to \Omega_5{}^{(0,0,1,1)}[t, x_1, x_2, x_3] - \Omega_8{}^{(0,0,2,0)}[t, x_1, x_2, x_3] - \Omega_2{}^{(0,1,0,1)}[t, x_1, x_2, x_3] + \Omega_6{}^{(0,1,1,0)}[t, x_1, x_2, x_3],$$
$$T[12][t, x_1, x_2, x_3] \to \Omega_9{}^{(0,0,1,1)}[t, x_1, x_2, x_3] - \Omega_7{}^{(0,0,2,0)}[t, x_1, x_2, x_3] - \Omega_2{}^{(1,0,0,1)}[t, x_1, x_2, x_3] + \Omega_6{}^{(1,0,1,0)}[t, x_1, x_2, x_3],$$
$$T[13][t, x_1, x_2, x_3] \to -\Omega_{10}{}^{(0,0,2,0)}[t, x_1, x_2, x_3] + \Omega_9{}^{(0,1,1,0)}[t, x_1, x_2, x_3] + \Omega_5{}^{(1,0,1,0)}[t, x_1, x_2, x_3] - \Omega_2{}^{(1,1,0,0)}[t, x_1, x_2, x_3],$$
$$T[14][t, x_1, x_2, x_3] \to -\Omega_4{}^{(0,0,2,0)}[t, x_1, x_2, x_3] + 2\,\Omega_9{}^{(1,0,1,0)}[t, x_1, x_2, x_3] - \Omega_2{}^{(2,0,0,0)}[t, x_1, x_2, x_3],\ T[15][t, x_1, x_2, x_3] \to \Omega_1{}^{(0,0,0,2)}[t, x_1, x_2, x_3] - 2\,\Omega_8{}^{(0,1,0,1)}[t, x_1, x_2, x_3] + \Omega_3{}^{(0,2,0,0)}[t, x_1, x_2, x_3],$$
$$T[16][t, x_1, x_2, x_3] \to -\Omega_1{}^{(0,0,1,1)}[t, x_1, x_2, x_3] + \Omega_5{}^{(0,1,0,1)}[t, x_1, x_2, x_3] + \Omega_8{}^{(0,1,1,0)}[t, x_1, x_2, x_3] - \Omega_6{}^{(0,2,0,0)}[t, x_1, x_2, x_3],\ T[17][t, x_1, x_2, x_3] \to -\Omega_1{}^{(0,0,2,0)}[t, x_1, x_2, x_3] + 2\,\Omega_5{}^{(0,1,1,0)}[t, x_1, x_2, x_3] - \Omega_2{}^{(0,2,0,0)}[t, x_1, x_2, x_3],$$

$$T[18][t, x_1, x_2, x_3] \to -\Omega_{10}{}^{(0,1,0,1)}[t, x_1, x_2, x_3] + \Omega_7{}^{(0,2,0,0)}[t, x_1, x_2, x_3] + \Omega_1{}^{(1,0,0,1)}[t, x_1, x_2, x_3] - \Omega_8{}^{(1,1,0,0)}[t, x_1, x_2, x_3],$$
$$T[19][t, x_1, x_2, x_3] \to \Omega_{10}{}^{(0,1,1,0)}[t, x_1, x_2, x_3] - \Omega_9{}^{(0,2,0,0)}[t, x_1, x_2, x_3] - \Omega_1{}^{(1,0,1,0)}[t, x_1, x_2, x_3] + \Omega_5{}^{(1,1,0,0)}[t, x_1, x_2, x_3],\ T[20][t, x_1, x_2, x_3] \to \Omega_4{}^{(0,2,0,0)}[t, x_1, x_2, x_3] - 2\,\Omega_{10}{}^{(1,1,0,0)}[t, x_1, x_2, x_3] + \Omega_1{}^{(2,0,0,0)}[t, x_1, x_2, x_3]\},$$

$$\{-T[1]^{(0,0,0,2)}[t, x_1, x_2, x_3] - T[1]^{(0,0,2,0)}[t, x_1, x_2, x_3] - T[1]^{(0,2,0,0)}[t, x_1, x_2, x_3] + T[1]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$-T[2]^{(0,0,0,2)}[t, x_1, x_2, x_3] - T[2]^{(0,0,2,0)}[t, x_1, x_2, x_3] - T[2]^{(0,2,0,0)}[t, x_1, x_2, x_3] + T[2]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$-T[3]^{(0,0,0,2)}[t, x_1, x_2, x_3] - T[3]^{(0,0,2,0)}[t, x_1, x_2, x_3] - T[3]^{(0,2,0,0)}[t, x_1, x_2, x_3] + T[3]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$T[4]^{(0,0,0,2)}[t, x_1, x_2, x_3] + T[4]^{(0,0,2,0)}[t, x_1, x_2, x_3] + T[4]^{(0,2,0,0)}[t, x_1, x_2, x_3] - T[4]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$-T[5]^{(0,0,0,2)}[t, x_1, x_2, x_3] - T[5]^{(0,0,2,0)}[t, x_1, x_2, x_3] - T[5]^{(0,2,0,0)}[t, x_1, x_2, x_3] + T[5]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$-T[6]^{(0,0,0,2)}[t, x_1, x_2, x_3] - T[6]^{(0,0,2,0)}[t, x_1, x_2, x_3] - T[6]^{(0,2,0,0)}[t, x_1, x_2, x_3] + T[6]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$-T[7]^{(0,0,0,2)}[t, x_1, x_2, x_3] - T[7]^{(0,0,2,0)}[t, x_1, x_2, x_3] - T[7]^{(0,2,0,0)}[t, x_1, x_2, x_3] + T[7]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$T[8]^{(0,0,0,2)}[t, x_1, x_2, x_3] + T[8]^{(0,0,2,0)}[t, x_1, x_2, x_3] + T[8]^{(0,2,0,0)}[t, x_1, x_2, x_3] - T[8]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$T[9]^{(0,0,0,2)}[t, x_1, x_2, x_3] + T[9]^{(0,0,2,0)}[t, x_1, x_2, x_3] + T[9]^{(0,2,0,0)}[t, x_1, x_2, x_3] - T[9]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$-T[10]^{(0,0,0,2)}[t, x_1, x_2, x_3] - T[10]^{(0,0,2,0)}[t, x_1, x_2, x_3] - T[10]^{(0,2,0,0)}[t, x_1, x_2, x_3] + T[10]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$T[11]^{(0,0,0,2)}[t, x_1, x_2, x_3] + T[11]^{(0,0,2,0)}[t, x_1, x_2, x_3] + T[11]^{(0,2,0,0)}[t, x_1, x_2, x_3] - T[11]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$-T[12]^{(0,0,0,2)}[t, x_1, x_2, x_3] - T[12]^{(0,0,2,0)}[t, x_1, x_2, x_3] - T[12]^{(0,2,0,0)}[t, x_1, x_2, x_3] + T[12]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$-T[13]^{(0,0,0,2)}[t, x_1, x_2, x_3] - T[13]^{(0,0,2,0)}[t, x_1, x_2, x_3] - T[13]^{(0,2,0,0)}[t, x_1, x_2, x_3] + T[13]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$T[14]^{(0,0,0,2)}[t, x_1, x_2, x_3] + T[14]^{(0,0,2,0)}[t, x_1, x_2, x_3] + T[14]^{(0,2,0,0)}[t, x_1, x_2, x_3] - T[14]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$T[15]^{(0,0,0,2)}[t, x_1, x_2, x_3] + T[15]^{(0,0,2,0)}[t, x_1, x_2, x_3] + T[15]^{(0,2,0,0)}[t, x_1, x_2, x_3] - T[15]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$T[16]^{(0,0,0,2)}[t, x_1, x_2, x_3] + T[16]^{(0,0,2,0)}[t, x_1, x_2, x_3] + T[16]^{(0,2,0,0)}[t, x_1, x_2, x_3] - T[16]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$-T[17]^{(0,0,0,2)}[t, x_1, x_2, x_3] - T[17]^{(0,0,2,0)}[t, x_1, x_2, x_3] - T[17]^{(0,2,0,0)}[t, x_1, x_2, x_3] + T[17]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$-T[18]^{(0,0,0,2)}[t, x_1, x_2, x_3] - T[18]^{(0,0,2,0)}[t, x_1, x_2, x_3] - T[18]^{(0,2,0,0)}[t, x_1, x_2, x_3] + T[18]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$-T[19]^{(0,0,0,2)}[t, x_1, x_2, x_3] - T[19]^{(0,0,2,0)}[t, x_1, x_2, x_3] - T[19]^{(0,2,0,0)}[t, x_1, x_2, x_3] + T[19]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0,$$
$$T[20]^{(0,0,0,2)}[t, x_1, x_2, x_3] + T[20]^{(0,0,2,0)}[t, x_1, x_2, x_3] + T[20]^{(0,2,0,0)}[t, x_1, x_2, x_3] - T[20]^{(2,0,0,0)}[t, x_1, x_2, x_3] == 0\}\}$$

Finally we note that the autonomous elements T[i]'s satisfy the same equation ($D_t^2 - D_{x_1}^2 - D_{x_2}^2 - D_{x_3}^2$) T[i] = 0 (D'Alembert's equation).