New Generic Attacks on Hash-based MACs

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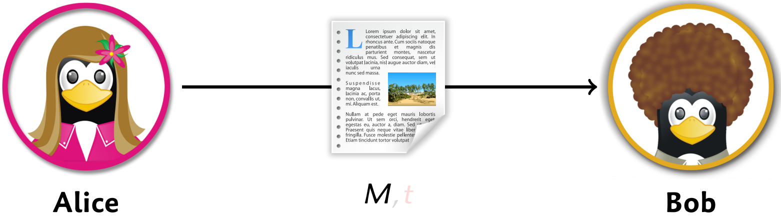
Message Authentication Codes



- Alice sends a message to Bob
- Bob wants to authenticate the message.
- Alice use a key k to compute a tag: $t = \mathrm{MAC}_k(M)$
- Bob verifies the tag with the same key k: $t \overset{?}{=} \mathrm{MAC}_k(M)$
- Symmetric equivalent to digital signatures

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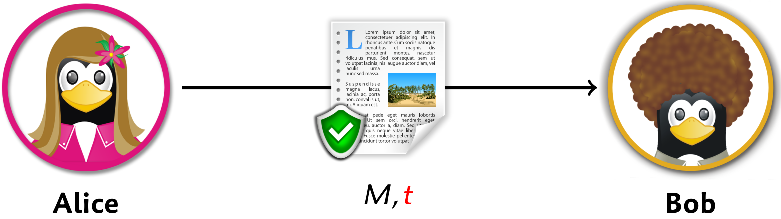

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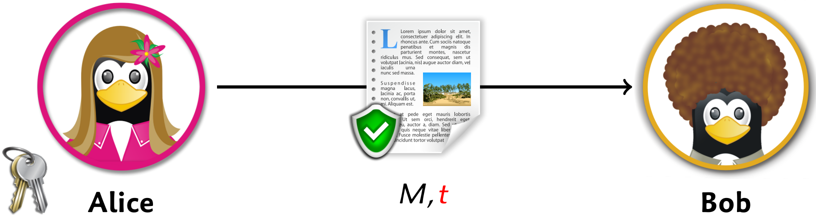

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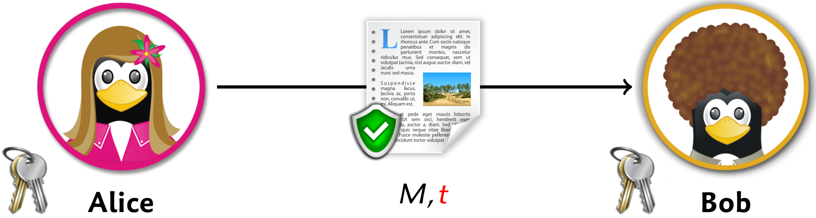

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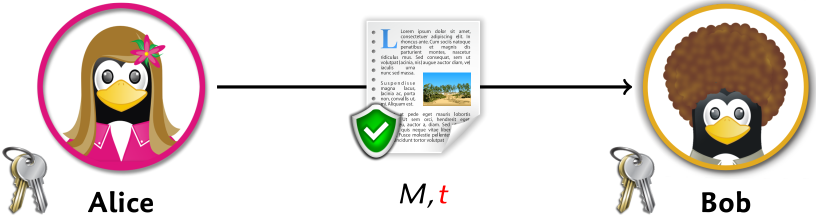

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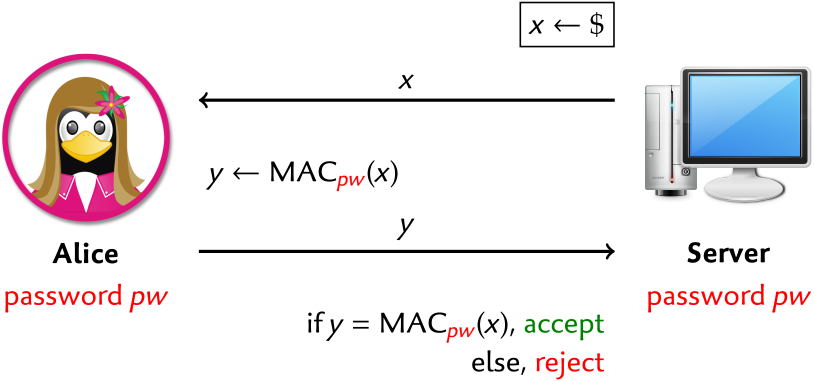
Example use: challenge-response authentication



$\boxed{x \leftarrow \$}$

x

$y \leftarrow \mathrm{MAC}_{pw}(x)$

y

Alice

password pw

Server

password pw

if $y = \mathrm{MAC}_{pw}(x)$, accept
else, reject

- CRAM-MD5 authentication in SASL, POP3, IMAP, SMTP, ...

MAC Constructions

- Dedicated designs
    - Pelican-MAC, SQUASH, SipHash
- From universal hash functions
    - UMAC, VMAC, Poly1305
- From block ciphers
    - CBC-MAC, OMAC, PMAC
- From hash functions
    - HMAC, Sandwich-MAC, Envelope-MAC

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Hash-based MACs (I)

- Secret-prefix MAC: $\mathrm{MAC}_k(M) = H(k \,\|\, M)$
  - Insecure with MD/SHA: length-extension attack
  - Compute $\mathrm{MAC}_k(M \,\|\, P)$ from $\mathrm{MAC}_k(M)$ without the key
- Secret-suffix MAC: $\mathrm{MAC}_k(M) = H(M \,\|\, k)$
  - Can be broken using offline collisions
- Use the key at the beginning and at the end
  - Sandwich-MAC: $H(k_1 \,\|\, M \,\|\, k_2)$
  - NMAC: $H(k_2 \,\|\, H(k_1 \,\|\, M))$
  - HMAC: $H((k \oplus \mathtt{opad}) \,\|\, H((k \oplus \mathtt{ipad}) \,\|\, M))$
  - Security proofs

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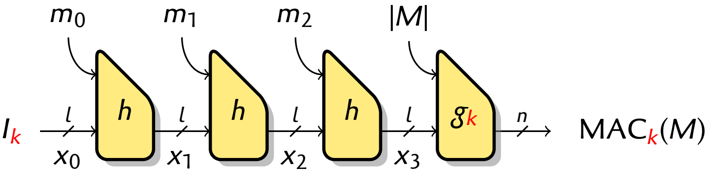


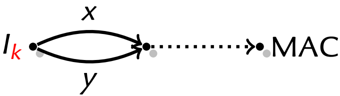
- l-bit chaining value
- n-bit output
- k-bit key

- Key-dependant initial value I_k
- Unkeyed compression function h
- Key-dependant finalization, with message length g_k

Security notions

- Key-recovery: given access to a MAC oracle, extract the key
- Forgery: given access to a MAC oracle, forge a valid pair
  - For a message chosen by the adversary: existential forgery
  - For a challenge given to the adversary: universal forgery
- Distinguishing games for hash-based MACs:
  - Distinguish $\mathrm{MAC}_k^{\mathcal{H}}$ from a PRF: distinguishing-R
    e.g. distinguish HMAC from a PRF
  - Distinguish $\mathrm{MAC}_k^{\mathcal{H}}$ from $\mathrm{MAC}_k^{\mathrm{PRF}}$: distinguishing-H
    e.g. distinguish HMAC-SHA1 from HMAC-PRF

Generic Attack on Hash-based MACs



1. Find internal collisions
    - Query $2^{l/2}$ 1-block messages
    - 1 internal collision expected, detected in the output
2. Query $t = \mathrm{MAC}(x \,\|\, m)$
3. $\left(y \,\|\, m, t\right)$ is a forgery

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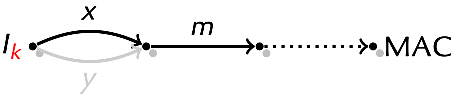

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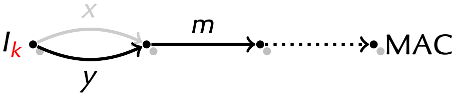

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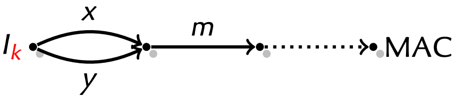

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    - Query $2^{l/2}$ 1-block messages
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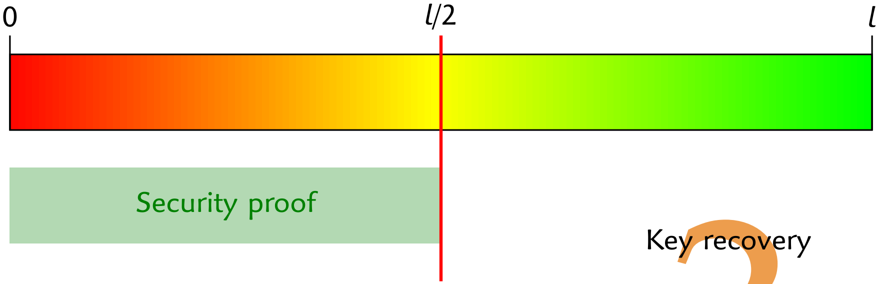
2. Query $t = \mathrm{MAC}(x \| m)$ and $t' = \mathrm{MAC}(y \| m)$
3. If $t = t'$ the oracle is a hash-based MAC: distinguishing-R

Security of hash-based MACS

With $n = l = k$:



Security proof

Key recovery

Existential forgery

Universal forgery

Distinguishing-R

Distinguishing-H

Outline

Introduction
- MACs
- Generic Attacks

New attacks
- Cycle detection
- Distinguishing-H attack
- State recovery attack

Key-recovery Attack on HMAC-GOST
- GOST
- HMAC-GOST

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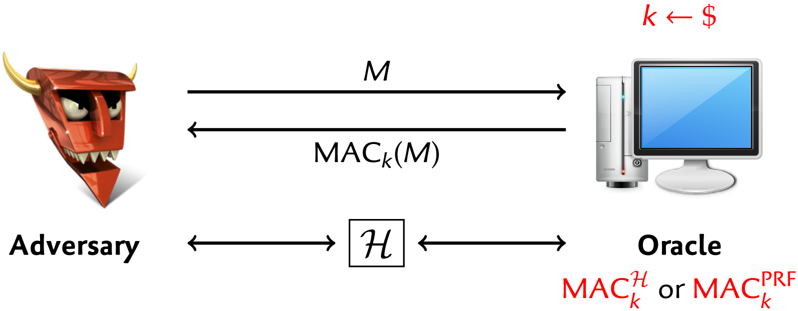
Distinguishing-H attack



$k \leftarrow \$$

M

$\mathrm{MAC}_k(M)$

Adversary

$\mathcal{H}$

Oracle

$\mathrm{MAC}_k^{\mathcal{H}}$ or $\mathrm{MAC}_k^{\mathrm{PRF}}$

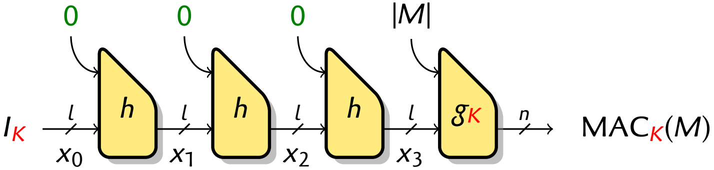
- Security notion from PRF
- Distinguish HMAC-SHA-1 from HMAC with a PRF

Distinguishing-H attack

- Collision-based attack does not work:
  - Any compression function has collisions
  - Secret key prevents pre-computed collision
- Common assumption: distinguishing-H attack should require 2^l

“If we can recognize the hash function inside HMAC,
it’s a bad hash function”

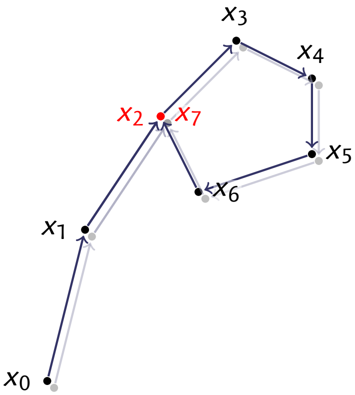
Main Idea



- Using a fixed message block, we iterate a fixed function
- Starting point and ending point unknown because of the key
- Can we still detect properties of the function $h_0 : x \mapsto h(x, 0)$?
  - Study the cycle structure of random mappings
  - Used to attack HMAC in related-key setting

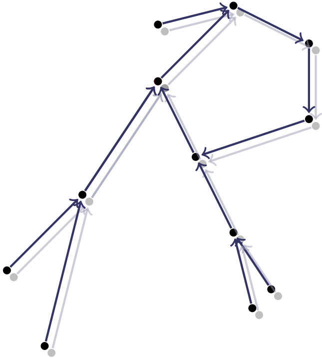
[Peyrin, Sasaki & Wang, Asiacrypt 12]

Random Mappings



- Functional graph of a random mapping $x \longrightarrow f(x)$
- Iterate f: $x_i = f(x_{i-1})$
- Collision after $\approx 2^{n/2}$ iterations
  - Cycles
- Trees rooted in the cycle
- Several components

Random Mappings



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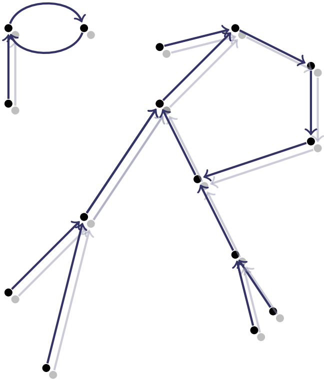

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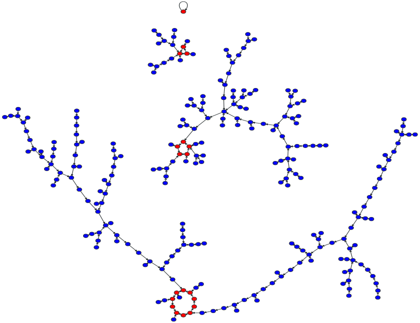
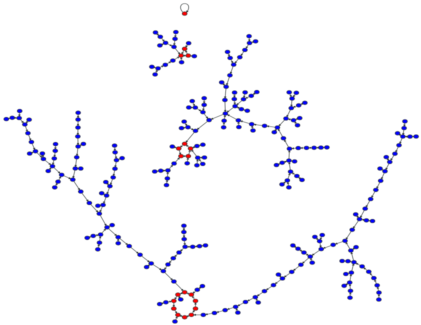
Cycle structure



Expected properties of a random mapping over N points:

- # Components: $\frac{1}{2}\log N$
- # Cyclic nodes: $\sqrt{\pi N/2}$
- Tail length: $\sqrt{\pi N/8}$
- Rho length: $\sqrt{\pi N/2}$
- Largest tree: $0.48N$
- Largest component: $0.76N$

Cycle structure



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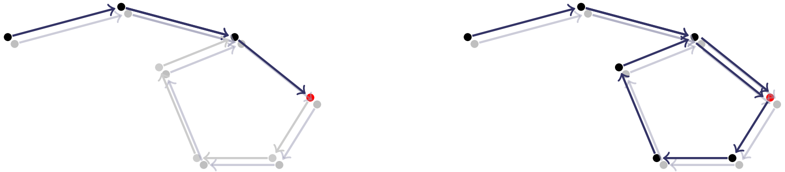
Using the cycle length

1. **Offline**: find the cycle length L of the main component of h_0
2. **Online**: query $t = \mathrm{MAC}(r \parallel [0]^{2^{\ell/2}})$ and $t' = \mathrm{MAC}(r \parallel [0]^{2^{\ell/2}+L})$



Success if

- The starting point is in the main component $p = 0.76$
- The cycle is reached with less than $2^{\ell/2}$ iterations $p \geq 0.5$

Randomize starting point

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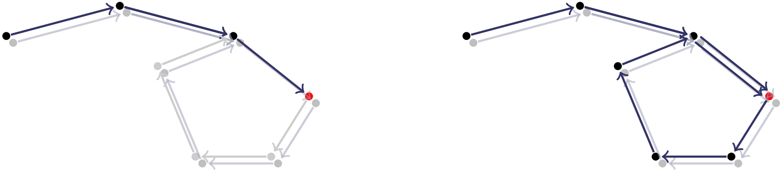

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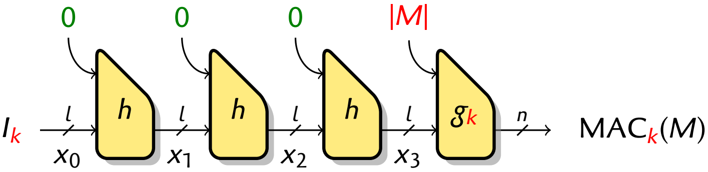
Randomize starting point

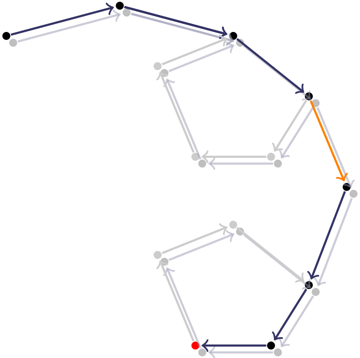
Dealing with the message length

Problem: most MACs use the message length.



Dealing with the message length

Solution: reach the cycle twice



$$M = r \parallel [0]^{2^{\ell/2}} \parallel [1] \parallel [0]^{2^{\ell/2}}$$

Dealing with the message length

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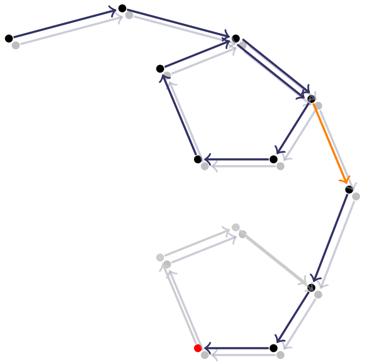
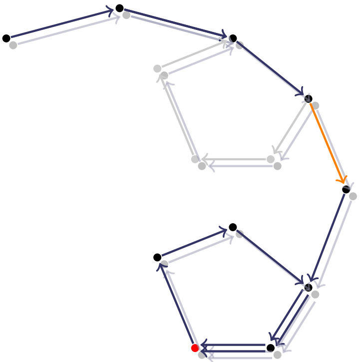


$M_1 = r \,\|\, [0]^{2^{\ell/2}+L} \,\|\, [1] \,\|\, [0]^{2^{\ell/2}}$



$M_2 = r \,\|\, [0]^{2^{\ell/2}} \,\|\, [1] \,\|\, [0]^{2^{\ell/2}+L}$

Distinguishing-H attack

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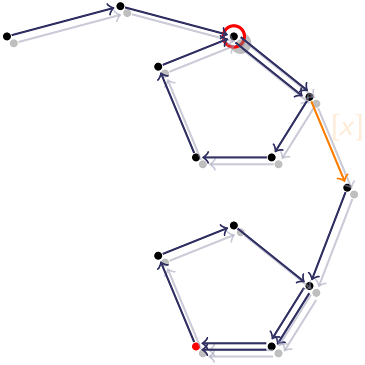
2. Online: query
$$t = \mathrm{MAC}(r \,\|\, [0]^{2^{\ell/2}} \,\|\, [1] \,\|\, [0]^{2^{\ell/2}+L})$$
$$t' = \mathrm{MAC}(r \,\|\, [0]^{2^{\ell/2}+L} \,\|\, [1] \,\|\, [0]^{2^{\ell/2}})$$
3. If $t = t'$, then h is the compression function in the oracle

Analysis

- Complexity: $2^{\ell/2+3}$ compression function calls
- Success probability: $p \simeq 0.14$
  - Both starting point are in the main component $p = 0.76^2$
  - Both cycles are reached with less than $2^{\ell/2}$ iterations $p \geq 0.5^2$

State recovery attack



- With high pr., first cyclic point is the root of the giant tree
- Binary search for first cyclic point

1. Query with several x:

$$t = \mathrm{MAC}(r \,\|\, [0]^{\alpha} \,\|\, [1] \,\|\, [0]^{2^{\ell/2}+L})$$
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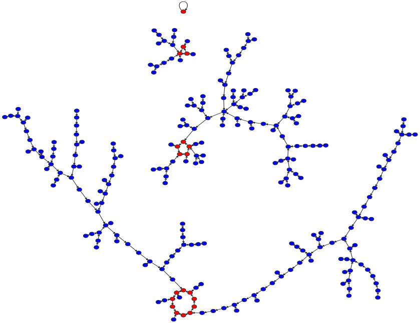
2. If $t = t'$ the cycle is reached with less than α steps

- Collision detection probabilistic: repeat with $\beta \log(\ell)$ messages

Cycle structure



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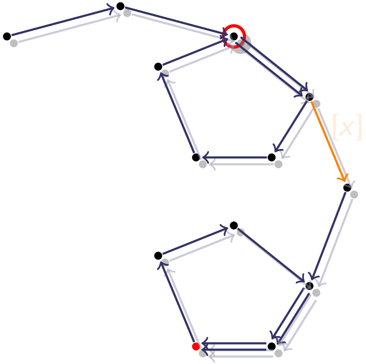

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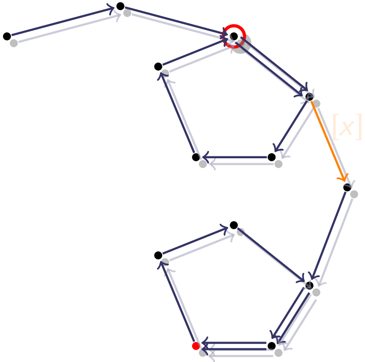

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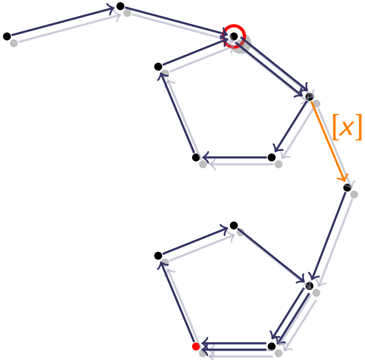

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2. If $t = t'$ the cycle is reached with less than α steps

- Collision detection probabilistic: repeat with $\beta \log(l)$ messages

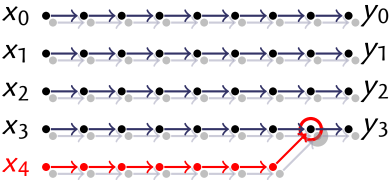
Variant with small messages

- Messages of length $2^{l/2}$ are not very practical...
  - SHA-1 and HAVAL limit the message length to 2^{64} bits
- Cycle detection impossible with messages shorter than $L \approx 2^{l/2}$

Compare with collision finding algorithms

- Pollard's rho algorithm use cycle detection
- Parallel collision search for van Oorschot and Wiener uses shorter chains

Collision finding with small chains



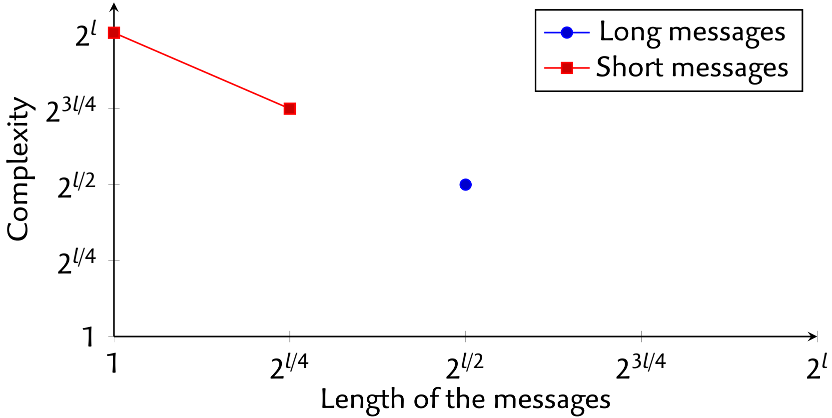
1. Compute chains $x \rightsquigarrow y$
   Stop when y distinguished
2. If $y \in \{y_i\}$, collision found

Using collisions for state recovery

- Collision points are not random
- Longer chains give more biased distribution
- Precompute collisions offline, and test online

Generic attacks on hash-based MACs

- Distinguishing-H and state recovery attacks
- Complexity 2^{l-s} with messages of length 2^s



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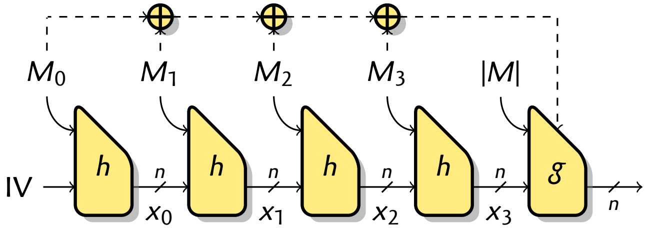
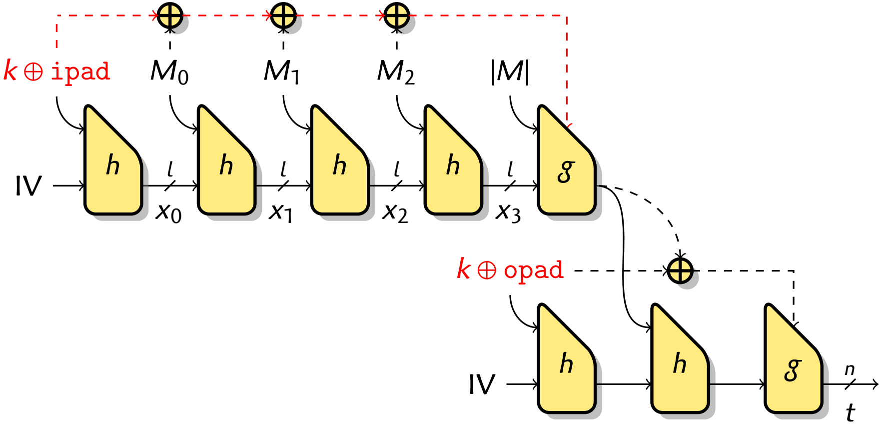
GOST



- Russian standard from 1994
- GOST and HMAC-GOST standardized by IETF
- $n = l = m = 256$

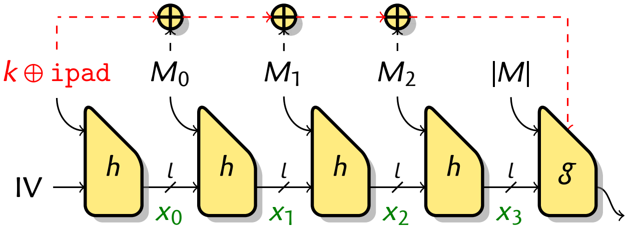
- Checksum (dashed lines)
  - Larger state should increase the security

HMAC-GOST



- In HMAC, key-dependant value used after the message
  - Related-key attacks on the last block

Key recovery attack



1. Recover the state
2. Build a multicollision: $2^{3\ell/4}$ messages with the same x_3
3. Query messages, detect collisions $g(x_3, k \oplus M) = g(x_3, k \oplus M')$
   Store $(M \oplus M', M)$ for $2^{\ell/2}$ collisions
4. Find collisions $g(x_3, x) = g(x_3, x')$ offline
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5. Detect match $M \oplus M' = x \oplus x'$. With high probability $M \oplus k = x$

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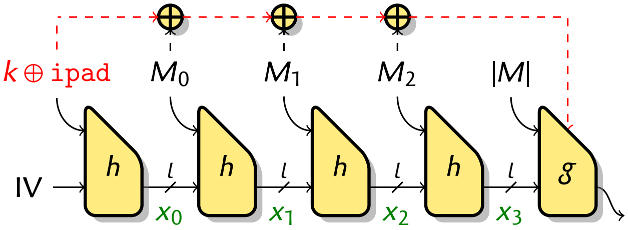

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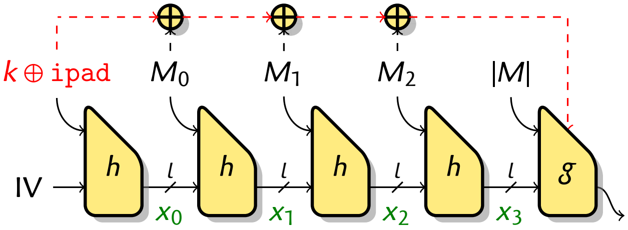

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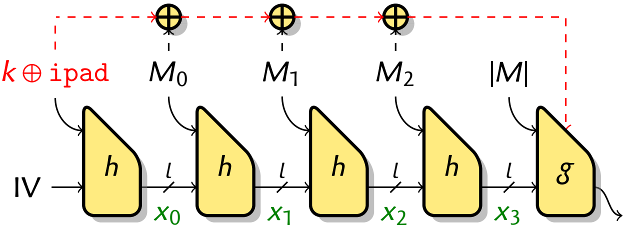

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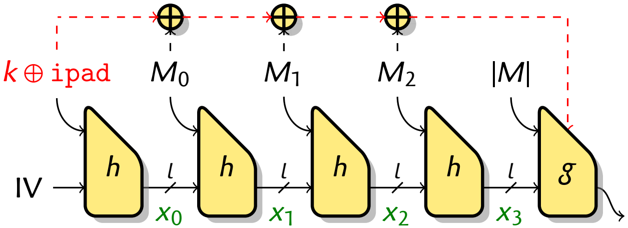

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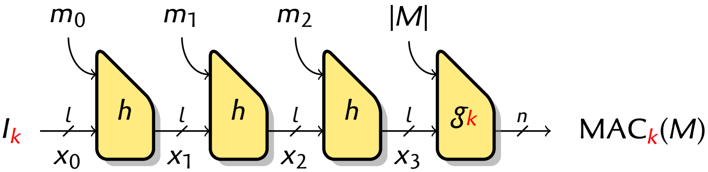
Conclusion



New generic attacks against hash-based MACs (single-key):

1. Distinguishing-H attack in $2^{\ell/2}$
    - State-recovery attack in $2^{\ell/2} \times \ell$
    - Not harder than distinguishing-R.

2. Key-recovery attack on HMAC-GOST in $2^{3\ell/4}$
    - Generic attack against hash functions with a checksum
    - The checksum weakens the design!

Thanks

Questions?

With the support of ERC project CRASH

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Comparison

Function	Attack	Complexity	M. len	Notes
`HMAC-MD5`	dist-H, st. rec.	2^{97}	2	
`HMAC-SHA-0`	dist-H	2^{100}	2	
`HMAC-HAVAL` (3-pass)	dist-H	2^{228}	2	
`HMAC-SHA-1` 62 mid. steps	dist-H	2^{157}	2	
Generic	dist-H, st. rec.	$\tilde{O}(2^{l/2})$	$2^{l/2}$	
	dist-H, st. rec.	$O(2^{l-s})$	2^s	$s \leq l/4$
Generic: checksum	key recovery	$O(2^{3l/4})$	$2^{l/4}$	
`HMAC-MD5`*	dist-H, st. rec.	2^{66}, 2^{78}	2^{64}	
		$O(2^{96})$	2^{32}	
`HMAC-HAVAL`§ (any)	dist-H, st. rec.	$\mathrm{O}(2^{202})$	2^{54}	
`HMAC-SHA-1`§	dist-H, st. rec.	$\mathrm{O}(2^{120})$	2^{40}	
`HMAC-GOST`*	key-recovery	2^{200}	2^{64}	

* `MD5`, `GOST`: arbitrary-length; § `SHA-1`, `HAVAL`: limited message length.