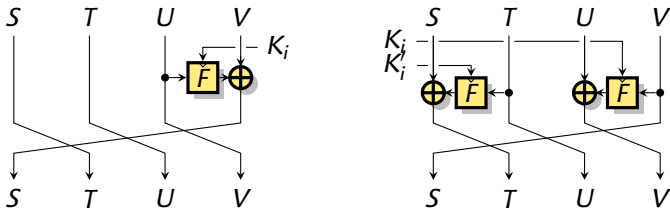


Attacks on Hash Functions based on Generalized Feistel Application to Lesamnta and SHAvite-3512

Charles Bouillaguet, Orr Dunkelman,
Pierre-Alain Fouque, Gaëtan Leurent

SAC 2010 – University of Waterloo

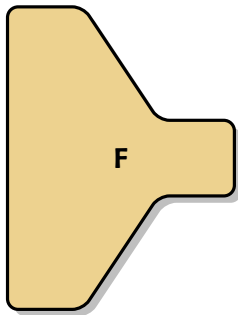


Hash Functions

- ▶ A public function with no structural properties.

- ▶ Cryptographic strength without keys!

▶ $F: \{0, 1\}^* \rightarrow \{0, 1\}^n$



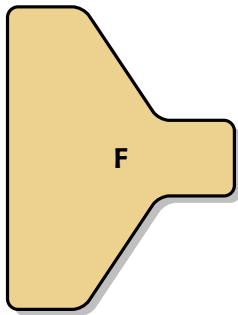
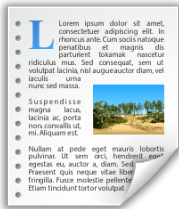
0x1d66ca77ab361c6f

Hash Functions

- ▶ A **public** function with **no structural properties**.

- ▶ Cryptographic strength without keys!

▶ $F: \{0, 1\}^* \rightarrow \{0, 1\}^n$



0x1d66ca77ab361c6f

The SHA-3 Competition

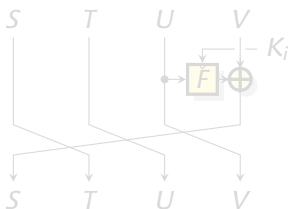
- ▶ Similar to the AES competition
- ▶ Organized by NIST

- ▶ Submission dead-line was October 2008: 64 candidates
- ▶ 51 valid submissions

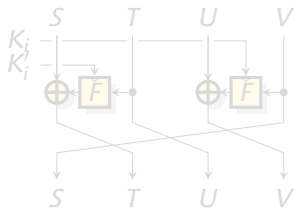
- ▶ 14 in the second round (July 2009)
- ▶ 5 finalists in September 2010?
- ▶ Winner in 2012?

Hash Function Design

- ▶ Hash function from a block cipher
 - ▶ Davies-Meyer, MMO, ...
- ▶ Block cipher from a fixed function
 - ▶ Feistel scheme
- ▶ Pick your favorite fixed function
 - ▶ AES?
- ▶ If the fixed function is too small, use a generalized Feistel:



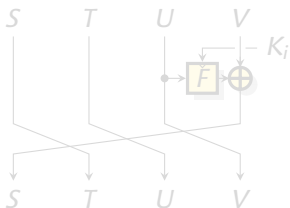
Lesamnta structure



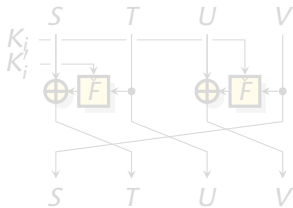
SHAvite-3₅₁₂ structure

Hash Function Design

- ▶ Hash function from a block cipher
 - ▶ Davies-Meyer, MMO, ...
- ▶ Block cipher from a fixed function
 - ▶ Feistel scheme
- ▶ Pick your favorite fixed function
 - ▶ AES?
- ▶ If the fixed function is too small, use a generalized Feistel:



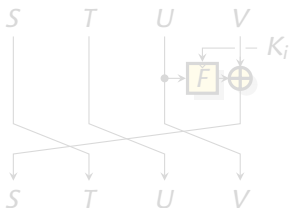
Lesamnta structure



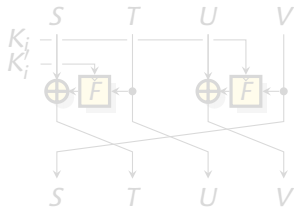
SHAvite-3₅₁₂ structure

Hash Function Design

- ▶ Hash function from a block cipher
 - ▶ Davies-Meyer, MMO, ...
- ▶ Block cipher from a fixed function
 - ▶ Feistel scheme
- ▶ Pick your favorite fixed function
 - ▶ AES?
- ▶ If the fixed function is too small, use a generalized Feistel:



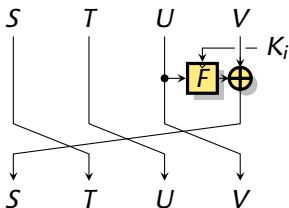
Lesamnta structure



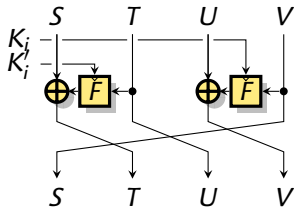
SHAvite-3512 structure

Hash Function Design

- ▶ Hash function from a block cipher
 - ▶ Davies-Meyer, MMO, ...
- ▶ Block cipher from a fixed function
 - ▶ Feistel scheme
- ▶ Pick your favorite fixed function
 - ▶ AES?
- ▶ If the fixed function is too small, use a generalized Feistel:



Lesamnta structure



SHAvite-3₅₁₂ structure

Feistel Design

- ▶ Ideal: each F_i is an independent ideal function/permutation
- ▶ In practice: $F_i(x) = F(k_i \oplus x)$ with a fixed F

Properties of $F_i(x) = F(k_i \oplus x)$

- (i) $\exists c_{ij} : \forall x, F_i(x \oplus c_{ij}) = F_j(x)$.
- (ii) $\forall \alpha, \#\{x : F_i(x) \oplus F_j(x) = \alpha\}$ is even
- (iii) $\bigoplus_x F_k(F_i(x) \oplus F_j(x)) = 0$

▶ $c_{ij} = k_i \oplus k_j$

Feistel Design

- ▶ Ideal: each F_i is an independent ideal function/permutation
- ▶ In practice: $F_i(x) = F(k_i \oplus x)$ with a **fixed** F

Properties of $F_i(x) = F(k_i \oplus x)$

- (i) $\exists c_{ij} : \forall x, F_i(x \oplus c_{ij}) = F_j(x)$.
- (ii) $\forall \alpha, \#\{x : F_i(x) \oplus F_j(x) = \alpha\}$ is even
- (iii) $\bigoplus_x F_k(F_i(x) \oplus F_j(x)) = 0$

▶ $c_{ij} = k_i \oplus k_j$

Feistel Design

- ▶ Ideal: each F_i is an independent ideal function/permutation
- ▶ In practice: $F_i(x) = F(k_i \oplus x)$ with a **fixed** F

Properties of $F_i(x) = F(k_i \oplus x)$

- (i) $\exists c_{ij} : \forall x, F_i(x \oplus c_{ij}) = F_j(x)$.
- (ii) $\forall \alpha, \#\{x : F_i(x) \oplus F_j(x) = \alpha\}$ is even
- (iii) $\bigoplus_x F_k(F_i(x) \oplus F_j(x)) = 0$

▶ $c_{ij} = k_i \oplus k_j$

Feistel Design

- ▶ Ideal: each F_i is an independent ideal function/permutation
- ▶ In practice: $F_i(x) = F(k_i \oplus x)$ with a **fixed** F

Properties of $F_i(x) = F(k_i \oplus x)$

- (i) $\exists c_{i,j} : \forall x, F_i(x \oplus c_{i,j}) = F_j(x)$.
- (ii) $\forall \alpha, \#\{x : F_i(x) \oplus F_j(x) = \alpha\}$ is even
- (iii) $\bigoplus_x F_k(F_i(x) \oplus F_j(x)) = 0$

- ▶ $c_{ij} = k_i \oplus k_j$

Cancellation Cryptanalysis

Main idea

Cancel the effect of the non-linear components
Using twice the same input pairs

- ▶ Generalized Feistel with slow diffusion
- ▶ $F_i(x) = F(k_i \oplus x)$
 - ▶ Can sometimes deal with more keys (see SHAvite-3₅₁₂)
- ▶ Hash function setting
 - ▶ Some results apply to block ciphers.

Cancellation Cryptanalysis

Main idea

Cancel the effect of the non-linear components
Using twice the same input pairs

- ▶ Generalized Feistel with slow diffusion
- ▶ $F_i(x) = F(k_i \oplus x)$
 - ▶ Can sometimes deal with more keys (see SHAvite-3₅₁₂)
- ▶ Hash function setting
 - ▶ Some results apply to block ciphers.

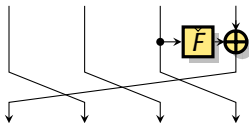
Cancellation Cryptanalysis

Main idea

Cancel the effect of the non-linear components
Using twice the same input pairs

- ▶ Generalized Feistel with slow diffusion
- ▶ $F_i(x) = F(k_i \oplus x)$
 - ▶ Can sometimes deal with more keys (see SHAvite-3₅₁₂)
- ▶ Hash function setting
 - ▶ Some results apply to block ciphers.

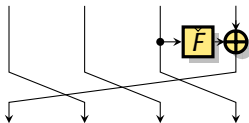
The Cancellation Property



i	S_i	T_i	U_i	V_i	
0	x	-	-	-	
1	-	x	-	-	
2	-	-	x	-	
3	y_1	-	-	x	$x \rightarrow y_1$
4	x	y_1	-	-	
5	-	x	y_1	-	
6	z	-	x	y_1	$y_1 \rightarrow z$
7	y'	z	-	x	$x \rightarrow y_2, y' = y_1 \oplus y_2$
8	x	y'	z	-	
9	w	x	y'	z	$z \rightarrow w$

- ▶ Full diffusion after 9 rounds
- ▶ If $y_1 = y_2 = y$, the differences cancel out
- ▶ Use constraints on the state

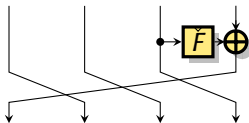
The Cancellation Property



i	S_i	T_i	U_i	V_i	
0	x	-	-	-	
1	-	x	-	-	
2	-	-	x	-	
3	y_1	-	-	x	$x \rightarrow y_1$
4	x	y_1	-	-	
5	-	x	y_1	-	
6	z	-	x	y_1	$y_1 \rightarrow z$
7	y'	z	-	x	$x \rightarrow y_2, y' = y_1 \oplus y_2$
8	x	y'	z	-	
9	w	x	y'	z	$z \rightarrow w$

- ▶ Full diffusion after 9 rounds
- ▶ If $y_1 = y_2 = y$, the differences cancel out
- ▶ Use constraints on the state

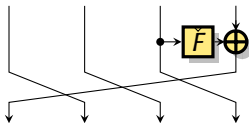
The Cancellation Property



i	S_i	T_i	U_i	V_i	
0	x	-	-	-	
1	-	x	-	-	
2	-	-	x	-	
3	y	-	-	x	$x \rightarrow y$
4	x	y	-	-	
5	-	x	y	-	
6	z	-	x	y	$y_1 \rightarrow z$
7	-	z	-	x	$x \rightarrow y$
8	x	-	z	-	
9	w	x	-	z	$z \rightarrow w$

- ▶ Full diffusion after 9 rounds
- ▶ If $y_1 = y_2 = y$, the differences cancel out
- ▶ Use constraints on the state

The Cancellation Property



i	S_i	T_i	U_i	V_i	
0	x	-	-	-	
1	-	x	-	-	
2	-	-	x	-	
3	y	-	-	x	$x \rightarrow y$
4	x	y	-	-	
5	-	x	y	-	
6	z	-	x	y	$y_1 \rightarrow z$
7	-	z	-	x	$x \rightarrow y$
8	x	-	z	-	
9	w	x	-	z	$z \rightarrow w$

- ▶ Full diffusion after 9 rounds
- ▶ If $y_1 = y_2 = y$, the differences cancel out
- ▶ Use constraints on the state

The Cancellation Property: Looking at the Values

We study values, starting at round 2:

i	S_i	T_i	U_i	V_i
2	a	b	c	d
3	$F_2(c) \oplus d$	a	b	c
4	$F_3(b) \oplus c$	$F_2(c) \oplus d$	a	b
5	$F_4(a) \oplus b$	$F_3(b) \oplus c$	$F_2(c) \oplus d$	a
6	$F_5(F_2(c) \oplus d) \oplus a$	$F_4(a) \oplus b$	$F_3(b) \oplus c$	$F_2(c) \oplus d$
7	$F_6(F_3(b) \oplus c)$ \oplus $F_2(c)$ $\oplus d$	$F_5(F_2(c) \oplus d) \oplus a$	$F_4(a) \oplus b$	$F_3(b) \oplus c$

Round 7: $F_6(F_3(b) \oplus c) \oplus F_2(c)$. They cancel if:

$$F_3(b) = c_{2,6} = K_2 \oplus K_6$$

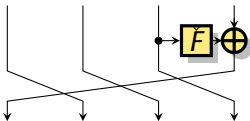
$$\text{i.e. } b = F_3^{-1}(K_2 \oplus K_6)$$

Attack Overview

- ▶ Partial preimage: Choose one part of the output
 - ▶ Gives preimage and collision attacks.
- ▶ Mostly generic in the round function.
- ▶ Hash function setting: no keys.

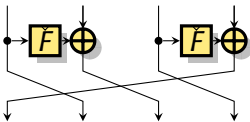
Result Overview

► Attacks on reduced *Lesamnta*



- 24 rounds out of 32: collision and preimage
- previous attacks: 16 rounds

► Attacks on reduced *SHAvite-3*₅₁₂



- 9 rounds out of 14: preimage
- previous attacks: 8 rounds

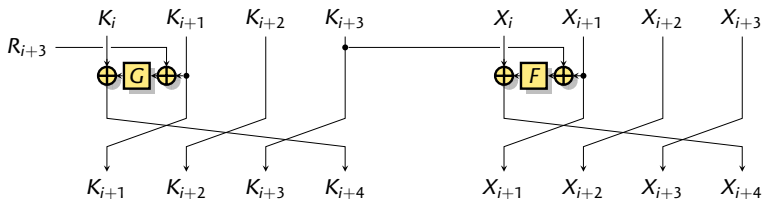
Lesamnta

- ▶ Merkle-Damgård with an MMO compression function
- ▶ Generalized Feistel
- ▶ Round function is AES-based



Shoichi Hirose, Hidenori Kuwakado, Hirotaka Yoshida
SHA-3 Proposal: Lesamnta
Submission to the NIST SHA-3 competition

Lesamnta (cont.)



$$X_{i+4} = X_i \oplus F(X_{i+1} \oplus K_{i+3})$$

$$K_{i+4} = K_i \oplus G(K_{i+1} \oplus R_{i+3}).$$

- ▶ Chaining value loaded to $K_{-3}, K_{-2}, K_{-1}, K_0$
- ▶ Message loaded to $X_{-3}, X_{-2}, X_{-1}, X_0$
- ▶ F and G AES-based

Lesamnta: Truncated Differential

i	S_i	T_i	U_i	V_i	
0	x	-	-	-	
1	-	x	-	-	
2	-	-	x	-	
\vdots		$(x \rightarrow x_1)$			
19	x_1	?	?	r	
20	?	x_1	?	?	
21	?	?	x_1	?	
22	?	?	?	x_1	
FF	?	?	?	x_1	

i	S_i	T_i	U_i	V_i	
2	-	-	x	-	
3	y	-	-	x	$x \rightarrow y$
4	x	y	-	-	
5	-	x	y	-	
6	z	-	x	y	$y \rightarrow z$
7	-	z	-	x	$x \rightarrow y$
8	x	-	z	-	
9	w	x	-	z	$z \rightarrow w$
10	z	w	x	-	
11	x_1	z	w	x	$x \rightarrow x_1$
12	r	x_1	z	w	$w \rightarrow x \oplus r$
13	-	r	x_1	z	$z \rightarrow w$
14	?	-	r	x_1	
15	$x_1 + t$?	-	r	$r \rightarrow t$
16	r	$x_1 + t$?	-	
17	?	r	$x_1 + t$?	
18	?	?	r	$x_1 + t$	
19	x_1	?	?	r	$r \rightarrow t$

Lesamnta: Truncated Differential

i	S_i	T_i	U_i	V_i
0	x	-	-	-
1	-	x	-	-
2	-	-	x	-
⋮		$(x \rightarrow x_1)$		
19	x_1	?	?	r
20	?	x_1	?	?
21	?	?	x_1	?
22	?	?	?	x_1
FF	?	?	?	x_1

Properties

- ▶ Using conditions on the state, **probability 1**.
- ▶ The transition $x \rightarrow x_1$ is **known**.

How to use it

- ▶ Start with a random message
- ▶ x_1 is the difference between the output and the target value
- ▶ Compute x from x_1
- ▶ Use $M + (x, 0, 0, 0)$

Lesamnta: Truncated Differential

i	S_i	T_i	U_i	V_i
0	x	-	-	-
1	-	x	-	-
2	-	-	x	-
\vdots		$(x \rightarrow x_1)$		
19	x_1	?	?	r
20	?	x_1	?	?
21	?	?	x_1	?
22	?	?	?	x_1
FF	?	?	?	x_1

Properties

- ▶ Using conditions on the state, **probability 1**.
- ▶ The transition $x \rightarrow x_1$ is **known**.

How to use it

- ▶ Start with a random message
- ▶ x_1 is the difference between the output and the target value
- ▶ Compute x from x_1
- ▶ Use $M + (x, 0, 0, 0)$

Lesamnta: Values

i	$X_i (= S_i)$
-1	d
0	c
1	b
2	a
3	$F_2(c) \oplus d$
4	$F_3(b) \oplus c$
5	$F_4(a) \oplus b$
6	$F_5(F_2(c) \oplus d) \oplus a$
7	$F_6(F_3(b) \oplus c) \oplus F_2(c) \oplus d$
8	$F_7(F_4(a) \oplus b) \oplus F_3(b) \oplus c$
9	$F_8(F_5(F_2(c) \oplus d) \oplus a) \oplus F_4(a) \oplus b$
10	$F_9(d) \oplus F_5(F_2(c) \oplus d) \oplus a$
11	$F_{10}(F_7(F_4(a) \oplus b) \oplus F_3(b) \oplus c) \oplus d$
12	$F_{11}(F_8(F_5(F_2(c) \oplus d) \oplus a) \oplus F_4(a) \oplus b) \oplus F_7(F_4(a) \oplus b) \oplus F_3(b) \oplus c$
13	$F_{12}(F_9(d) \oplus F_5(F_2(c) \oplus d) \oplus a)$ $\oplus F_8(F_5(F_2(c) \oplus d) \oplus a)$ $\oplus F_4(a) \oplus b$
15	$F_{14}(X_{12}) \oplus F_{10}(F_7(F_4(a) \oplus b) \oplus F_3(b) \oplus c) \oplus d$
16	$F_{15}(F_4(a) \oplus b) \oplus X_{12}$
19	$F_{18}(F_{15}(F_4(a) \oplus b) \oplus X_{12})$ $\oplus F_{14}(X_{12})$ $\oplus F_{10}(F_7(F_4(a) \oplus b) \oplus F_3(b) \oplus c) \oplus d$

Lesamnta Cancellation Conditions

Round 7: $F_6(F_3(b) \oplus c) \oplus F_2(c)$.

They cancel if: $F_3(b) = c_{2,6} = K_2 \oplus K_6$

i.e. $b = F_3^{-1}(K_2 \oplus K_6)$

Round 13: $F_{12}(F_9(d) \oplus F_5(F_2(c) \oplus d) \oplus a) \oplus F_8(F_5(F_2(c) \oplus d) \oplus a)$.

They cancel if: $F_9(d) = c_{8,12} = K_8 \oplus K_{12}$

i.e. $d = F_9^{-1}(K_8 \oplus K_{12})$

Round 19: $F_{18}(F_{15}(F_4(a) \oplus b) \oplus X_{12}) \oplus F_{14}(X_{12})$.

They cancel if: $F_{15}(F_4(a) \oplus b) = c_{14,18} = K_{14} \oplus K_{18}$

i.e. $a = F_4^{-1}(F_{15}^{-1}(K_{14} \oplus K_{18}) \oplus b)$

22-round Attacks

- ▶ Compute a, b, d , to satisfy the cancellation conditions.
- ▶ Set the state at round 2 to (a, b, c, d) .
- ▶ Express the output as a function of c
- ▶ $V_0 = \eta$
 - ▶ $\eta = b \oplus F_0(a \oplus F_3(d))$
- ▶ $V_{22} = F(c \oplus \alpha) \oplus \beta$
 - ▶ $\alpha = K_{11} \oplus F_8(F_5(a) \oplus b) \oplus F_4(b)$
 - ▶ $\beta = d$
- ▶ For a target value \bar{H} , set $c = F^{-1}(\bar{H} \oplus \eta \oplus \beta) \oplus \alpha$
- ▶ This gives $V_0 \oplus V_{22} = \bar{H}$

22-round Attacks

- ▶ Compute a, b, d , to satisfy the cancellation conditions.
- ▶ Set the state at round 2 to (a, b, c, d) .
- ▶ Express the output as a function of c
- ▶ $V_0 = \eta$
 - ▶ $\eta = b \oplus F_0(a \oplus F_3(d))$
- ▶ $V_{22} = F(c \oplus \alpha) \oplus \beta$
 - ▶ $\alpha = K_{11} \oplus F_8(F_5(a) \oplus b) \oplus F_4(b)$
 - ▶ $\beta = d$
- ▶ For a target value \bar{H} , set $c = F^{-1}(\bar{H} \oplus \eta \oplus \beta) \oplus \alpha$
- ▶ This gives $V_0 \oplus V_{22} = \bar{H}$

22-round Attacks

- ▶ Compute a, b, d , to satisfy the cancellation conditions.
- ▶ Set the state at round 2 to (a, b, c, d) .
- ▶ Express the output as a function of c
- ▶ $V_0 = \eta$
 - ▶ $\eta = b \oplus F_0(a \oplus F_3(d))$
- ▶ $V_{22} = F(c \oplus \alpha) \oplus \beta$
 - ▶ $\alpha = K_{11} \oplus F_8(F_5(a) \oplus b) \oplus F_4(b)$
 - ▶ $\beta = d$
- ▶ For a target value \bar{H} , set $c = F^{-1}(\bar{H} \oplus \eta \oplus \beta) \oplus \alpha$
- ▶ This gives $V_0 \oplus V_{22} = \bar{H}$

24-round Attacks

- ▶ Compute a, b, d , to satisfy the cancellation conditions.
- ▶ Set the state at round 4 to (a, b, c, d) .
- ▶ $V_0 = F(c \oplus \gamma) \oplus \lambda$
 - ▶ $\gamma = F_1(b \oplus F_2(a \oplus F_3(d)))$
 - ▶ $\lambda = d$
- ▶ $V_{24} = F(c \oplus \alpha) \oplus \beta$
 - ▶ $\alpha = K_{13} \oplus F_{10}(F_7(a) \oplus b) \oplus F_6(b)$
 - ▶ $\beta = d$
- ▶ The output is $H = F(c \oplus \gamma) \oplus F(c \oplus \alpha)$.
- ▶ To reach a target \bar{H} , we need a pair of values for F with
 - ▶ input difference $\alpha \oplus \gamma$
 - ▶ output difference \bar{H}
- ▶ We can store them in a table.

24-round Attacks

- ▶ Compute a, b, d , to satisfy the cancellation conditions.
- ▶ Set the state at round 4 to (a, b, c, d) .
- ▶ $V_0 = F(c \oplus \gamma) \oplus \lambda$
 - ▶ $\gamma = F_1(b \oplus F_2(a \oplus F_3(d)))$
 - ▶ $\lambda = d$
- ▶ $V_{24} = F(c \oplus \alpha) \oplus \beta$
 - ▶ $\alpha = K_{13} \oplus F_{10}(F_7(a) \oplus b) \oplus F_6(b)$
 - ▶ $\beta = d$
- ▶ The output is $H = F(c \oplus \gamma) \oplus F(c \oplus \alpha)$.
- ▶ To reach a target \bar{H} , we need a pair of values for F with
 - ▶ input difference $\alpha \oplus \gamma$
 - ▶ output difference \bar{H}
- ▶ We can store them in a table.

24-round Attacks

- ▶ Compute a, b, d , to satisfy the cancellation conditions.
- ▶ Set the state at round 4 to (a, b, c, d) .
- ▶ $V_0 = F(c \oplus \gamma) \oplus \lambda$
 - ▶ $\gamma = F_1(b \oplus F_2(a \oplus F_3(d)))$
 - ▶ $\lambda = d$
- ▶ $V_{24} = F(c \oplus \alpha) \oplus \beta$
 - ▶ $\alpha = K_{13} \oplus F_{10}(F_7(a) \oplus b) \oplus F_6(b)$
 - ▶ $\beta = d$
- ▶ The output is $H = F(c \oplus \gamma) \oplus F(c \oplus \alpha)$.
- ▶ To reach a target \bar{H} , we need a pair of values for F with
 - ▶ input difference $\alpha \oplus \gamma$
 - ▶ output difference \bar{H}
- ▶ We can store them in a table.

Improved 24-round Attack


- ▶ The output is $H = F(c \oplus \gamma) \oplus F(c \oplus \alpha)$.
- ▶ F is AES-based.
- ▶ Use the symmetry property of AES:
 - ▶ If x is symmetric, then $F(x)$ is symmetric
- ▶ Try random keys until $\alpha \oplus \gamma$ is symmetric
- ▶ For all symmetric u , $c = \alpha \oplus u$ gives a symmetric output
- ▶ One output word symmetric for an amortized cost of 1
 - ▶ $\approx n/8$ bits set to zero

Results: SHAvite-3512

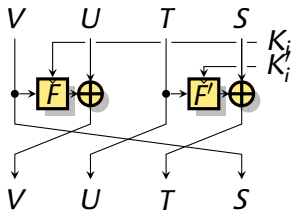
		<i>Lesamnta-256</i>		<i>Lesamnta-512</i>		
	Attack	Rounds	Time	Memory	Time	Memory
<i>Generic</i>	Collision	22	2^{96}	-	2^{192}	-
	2 nd Preimage	22	2^{192}	-	2^{384}	-
	Collision	24	2^{96}	2^{64}	2^{192}	2^{128}
	2 nd Preimage	24	2^{192}	2^{64}	2^{384}	2^{128}
<i>Specific</i>	Collision	24	2^{112}	-	2^{224}	-
	2 nd Preimage	24	2^{240}	-	N/A	

SHAvite-3₅₁₂

- ▶ Merkle-Damgård with a Davies-Meyer compression function
- ▶ Generalized Feistel
- ▶ Round function is AES-based

 Eli Biham and Orr Dunkelman
The SHAvite-3 Hash Function
Submission to the NIST SHA-3 competition

SHAvite-3₅₁₂ (cont.)



- ▶ 14 rounds
- ▶ Davies-Meyer (message is the key)
- ▶ $F_i(x) = AES(AES(AES(AES(x \oplus k_i^0) \oplus k_i^1) \oplus k_i^2) \oplus k_i^3)$
- ▶ F is one AES round.
- ▶ Key schedule mixes linear operations and AES rounds.

SHAvite-3512: Truncated Differential

i	S_i	T_i	U_i	V_i
0	?	x_2	?	x
1	x	-	x_2	x_1
2	x_1	x	-	-
3	-	-	x	-
4	-	-	-	x
5	x	-	-	y
6	y	x	-	z
7	z	-	x	w
8	w	z	-	?
9	?	-	z	?
FF	?	x_2	?	?

 $x_1 \rightarrow x_2$ $x \rightarrow x_1$ $x \rightarrow y$ $y \rightarrow z$ $x \rightarrow y, z \rightarrow w$ $z \rightarrow w$

Properties

- ▶ Using conditions on the state, **probability 1**.
- ▶ The transitions $x \rightarrow x_1$ and $x_1 \rightarrow x_2$ are **known**.
- ▶ Same attack as earlier.

Problem

- ▶ F has many keys

SHAvite-3512: Truncated Differential

i	S_i	T_i	U_i	V_i
0	?	x_2	?	x
1	x	-	x_2	x_1
2	x_1	x	-	-
3	-	-	x	-
4	-	-	-	x
5	x	-	-	y
6	y	x	-	z
7	z	-	x	w
8	w	z	-	?
9	?	-	z	?
FF	?	x_2	?	?

 $x_1 \rightarrow x_2$ $x \rightarrow x_1$ $x \rightarrow y$ $y \rightarrow z$ $x \rightarrow y, z \rightarrow w$ $z \rightarrow w$

Properties

- ▶ Using conditions on the state, **probability 1**.
- ▶ The transitions $x \rightarrow x_1$ and $x_1 \rightarrow x_2$ are **known**.
- ▶ Same attack as earlier.

Problem

- ▶ F has many keys

SHAvite-3₅₁₂: Values

i	X_i/Y_i
X_0	$b \oplus F_3(c) \oplus F'_1(c \oplus F_2(d \oplus F'_3(a)))$
Y_0	$d \oplus F'_3(a) \oplus F_1(a \oplus F'_2(b \oplus F_3(c)))$
X_1	$a \oplus F'_2(b \oplus F_3(c))$
Y_1	$c \oplus F_2(d \oplus F'_3(a))$
X_2	$d \oplus F'_3(a)$
Y_2	$b \oplus F_3(c)$
X_3	c
Y_3	a
X_4	b
Y_4	d
X_5	$a \oplus F_4(b)$
Y_5	$c \oplus F'_4(d)$
X_6	$d \oplus F_5(a \oplus F_4(b))$
Y_6	$b \oplus F'_5(c \oplus F'_4(d))$
X_7	$c \oplus F'_4(d) \oplus F_6(d \oplus F_5(a \oplus F_4(b)))$
Y_7	$a \oplus F_4(b) \oplus F'_6(b \oplus F'_5(c \oplus F'_4(d)))$
X_8	$b \oplus F'_5(c \oplus F'_4(d)) \oplus F_7(c)$
Y_8	$d \oplus F_5(a \oplus F_4(b)) \oplus F'_7(a \oplus F_4(b) \oplus F'_6(b \oplus F'_5(c \oplus F'_4(d))))$
X_9	$a \oplus F_4(b) \oplus F'_6(b \oplus F'_5(c \oplus F'_4(d))) \oplus F_8(b \oplus F'_5(c \oplus F'_4(d)) \oplus F_7(c))$

Message Conditions: SHAvite-3512

Round 7 $F'_4(d) \oplus F_6(d \oplus F_5(a \oplus F_4(b)))$.

They cancel if: $F_5(a \oplus F_4(b)) = k_{1,4}^0 \oplus k_{0,6}^0$

and $(k_{1,4}^1, k_{1,4}^2, k_{1,4}^3) = (k_{0,6}^1, k_{0,6}^2, k_{0,6}^3)$.

Round 9 $F'_6(b \oplus F'_5(c \oplus F'_4(d))) \oplus F_8(b \oplus F'_5(c \oplus F'_4(d))) \oplus F_7(c)$.

They cancel if: $F_7(c) = k_{1,6}^0 \oplus k_{0,8}^0$

and $(k_{1,6}^1, k_{1,6}^2, k_{1,6}^3) = (k_{0,8}^1, k_{0,8}^2, k_{0,8}^3)$.

Message Conditions: SHAvite-3₅₁₂

Round 7 $F'_4(d) \oplus F_6(d \oplus F_5(a \oplus F_4(b)))$.

They cancel if: $F_5(a \oplus F_4(b)) = k_{1,4}^0 \oplus k_{0,6}^0$

and $(k_{1,4}^1, k_{1,4}^2, k_{1,4}^3) = (k_{0,6}^1, k_{0,6}^2, k_{0,6}^3)$.

Round 9 $F'_6(b \oplus F'_5(c \oplus F'_4(d))) \oplus F_8(b \oplus F'_5(c \oplus F'_4(d)) \oplus F_7(c))$.

They cancel if: $F_7(c) = k_{1,6}^0 \oplus k_{0,8}^0$

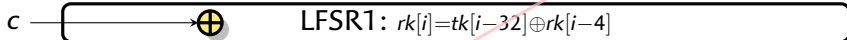
and $(k_{1,6}^1, k_{1,6}^2, k_{1,6}^3) = (k_{0,8}^1, k_{0,8}^2, k_{0,8}^3)$.

Message Expansion

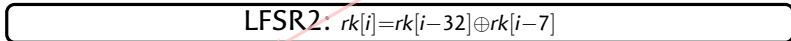
$rk[128\dots 131, 132\dots 135, 136\dots 139, 140\dots 143, 144\dots 147, 148\dots 151, 152\dots 155, 156\dots 159]$



$tk[128\dots 131, 132\dots 135, 136\dots 139, 140\dots 143, 144\dots 147, 148\dots 151, 152\dots 155, 156\dots 159]$



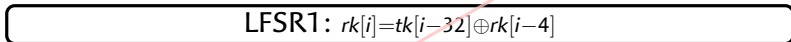
$rk[160\dots 163, 164\dots 167, 168\dots 171, 172\dots 175, 176\dots 179, 180\dots 183, 184\dots 187, 188\dots 191]$



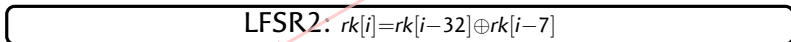
$rk[192\dots 195, 196\dots 199, 200\dots 203, 204\dots 207, 208\dots 211, 212\dots 215, 216\dots 219, 220\dots 223]$



$tk[192\dots 195, 196\dots 199, 200\dots 203, 204\dots 207, 208\dots 211, 212\dots 215, 216\dots 219, 220\dots 223]$



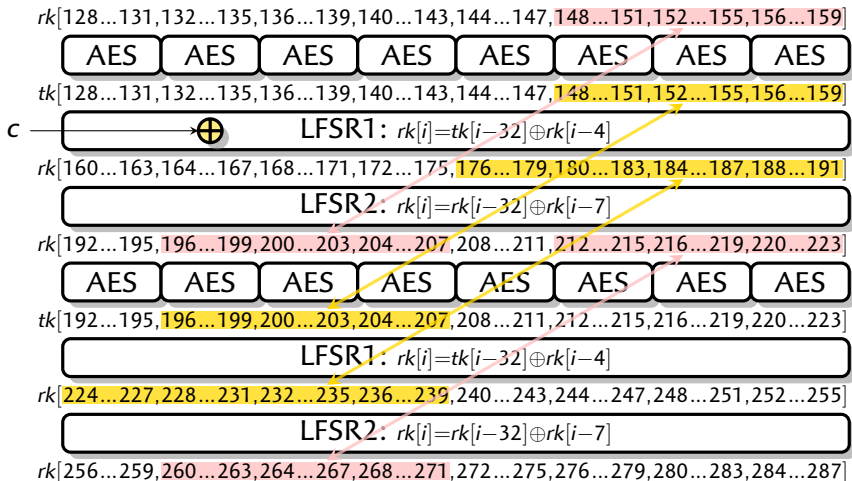
$rk[224\dots 227, 228\dots 231, 232\dots 235, 236\dots 239, 240\dots 243, 244\dots 247, 248\dots 251, 252\dots 255]$



$rk[256\dots 259, 260\dots 263, 264\dots 267, 268\dots 271, 272\dots 275, 276\dots 279, 280\dots 283, 284\dots 287]$

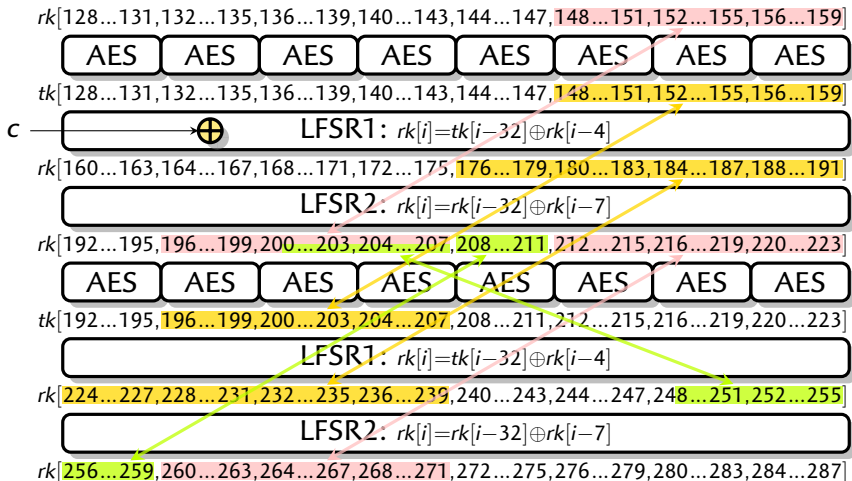
1 Propagate constraints

Message Expansion



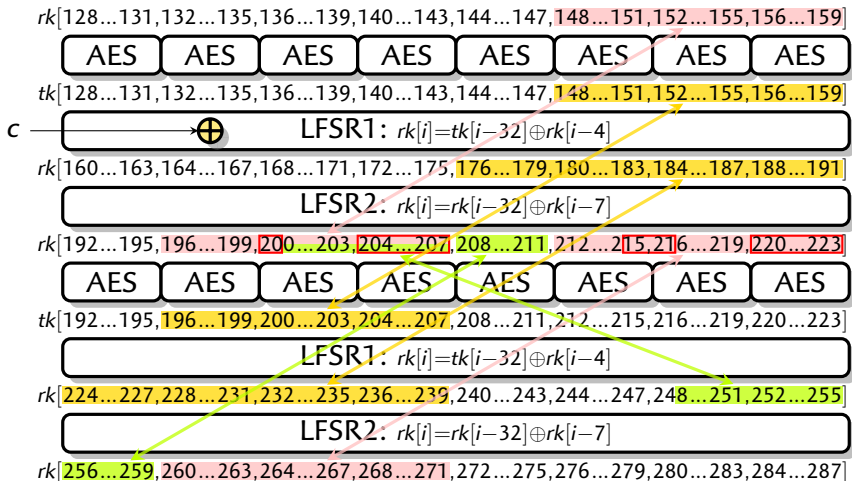
1 Propagate constraints

Message Expansion



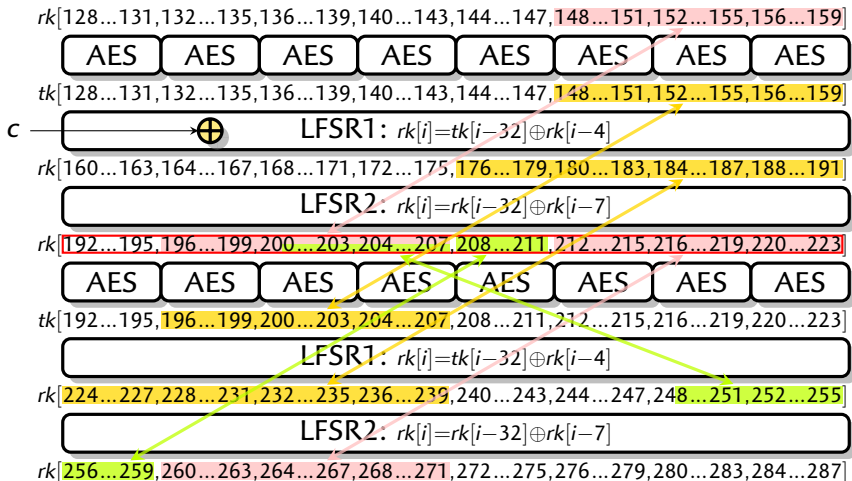
1 Propagate constraints

Message Expansion



2 Guess values

Message Expansion



3 Compute the missing values; check coherence

Solving the Conditions

- ▶ We can build a chaining value satisfying the 6 conditions with cost 2^{96} .
- ▶ Each chaining value can be used 2^{128} times to fix 128 bits of the output.
 - ▶ Cost of finding a good message is amortized.
- ▶ Attacks on 9-round *SHAvite-3*₅₁₂:
 - ▶ Free-start preimage with complexity 2^{384}
 - ▶ Second-Preimage with complexity 2^{448} .

Later Improvements

- ▶ 10-round attack using both degrees of freedom
- ▶ Pseudo-attacks on the full 14 rounds (chosen salts)



Praveen Gauravaram, Gaëtan Leurent, Florian Mendel,
María Naya-Plasencia, Thomas Peyrin, Christian Rechberger, and
Martin Schläffer

Cryptanalysis of the 10-Round Hash and Full Compression Function
of *SHAvite-3*₅₁₂

Africacrypt 2010

Results: SHAvite-3512

Attack	Rounds	Comp. Fun.		Hash Fun.	
		Time	Mem.	Time	Mem.
2 nd Preimage new	9	2^{384}	-	2^{448}	2^{64}
2 nd Preimage [AF'10]	10	2^{480}	-	2^{496}	2^{16}
2 nd Preimage improved	10	2^{448}	-	2^{480}	2^{32}
2 nd Preimage improved	10	2^{416}	2^{64}	2^{464}	2^{64}
2 nd Preimage improved	10	2^{384}	2^{128}	2^{448}	2^{128}
Collision ¹ [AF'10]	14	2^{192}	2^{128}	N/A	
Preimage ¹ [AF'10]	14	2^{384}	2^{128}	N/A	
Preimage ¹ [AF'10]	14	2^{448}	-	N/A	

¹ Chosen salt attacks

Conclusion

- ▶ Shows the difference an ideal Feistel with independent round functions and a practical construction.
- ▶ *Full version*: ePrint report 2009/634.
 - ▶ Includes some block cipher results
- ▶ Any questions?